

# Concrete

## Zama's FHE compiler

# Meet The Team



**Quentin Bourgerie**  
Head of Concrete



**Samuel Tap**  
Scientific Advisor

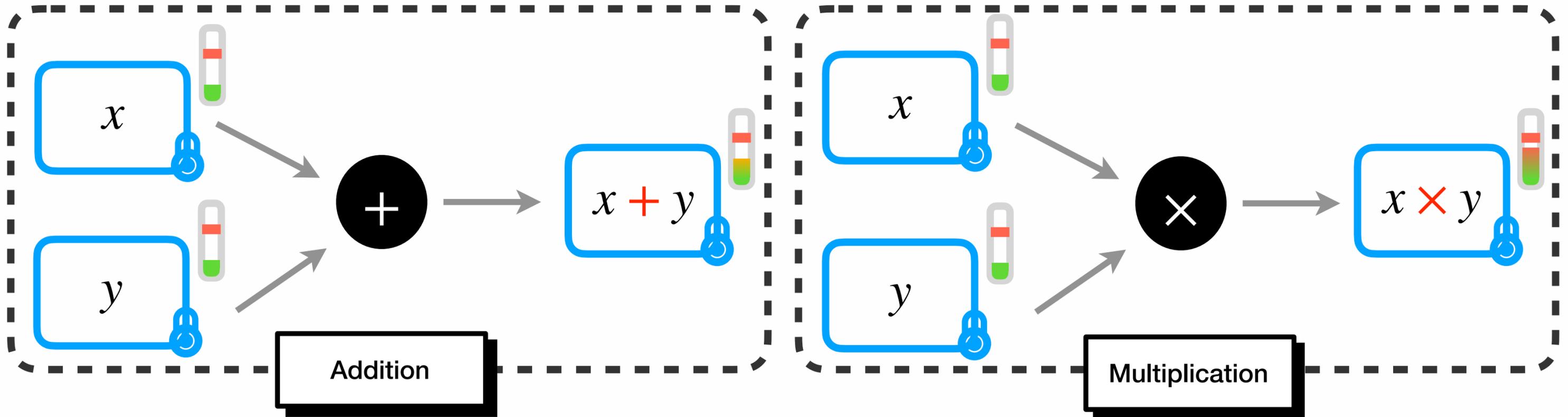
# Agenda

Introduction	04
Concrete Python	13
Concrete Compiler	20
Concrete Optimizer	27
Conclusion	36

# Introduction

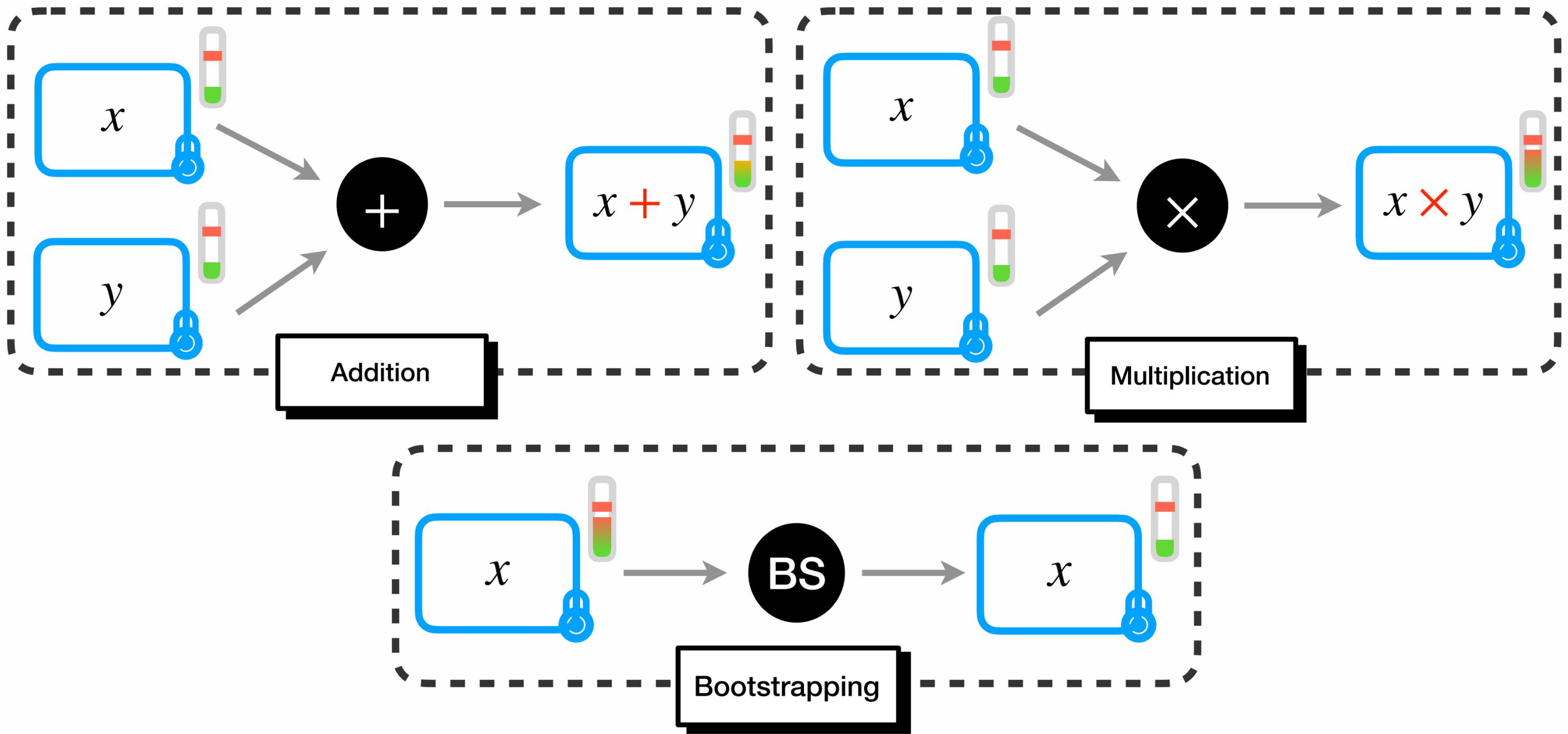
Why do we need a compiler for FHE ?

## FHE

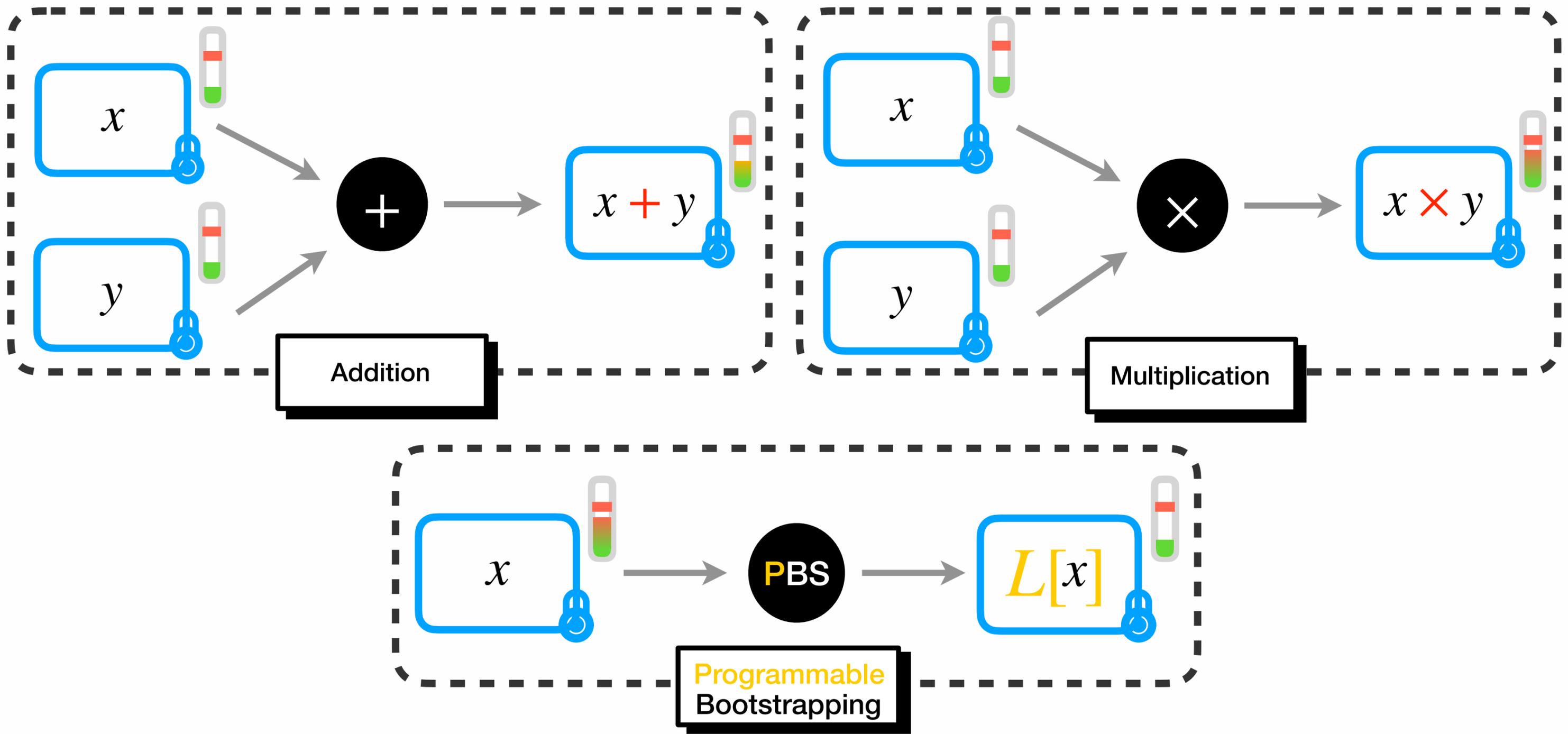


**too much noise  $\implies$  incorrect decryption** 🦴

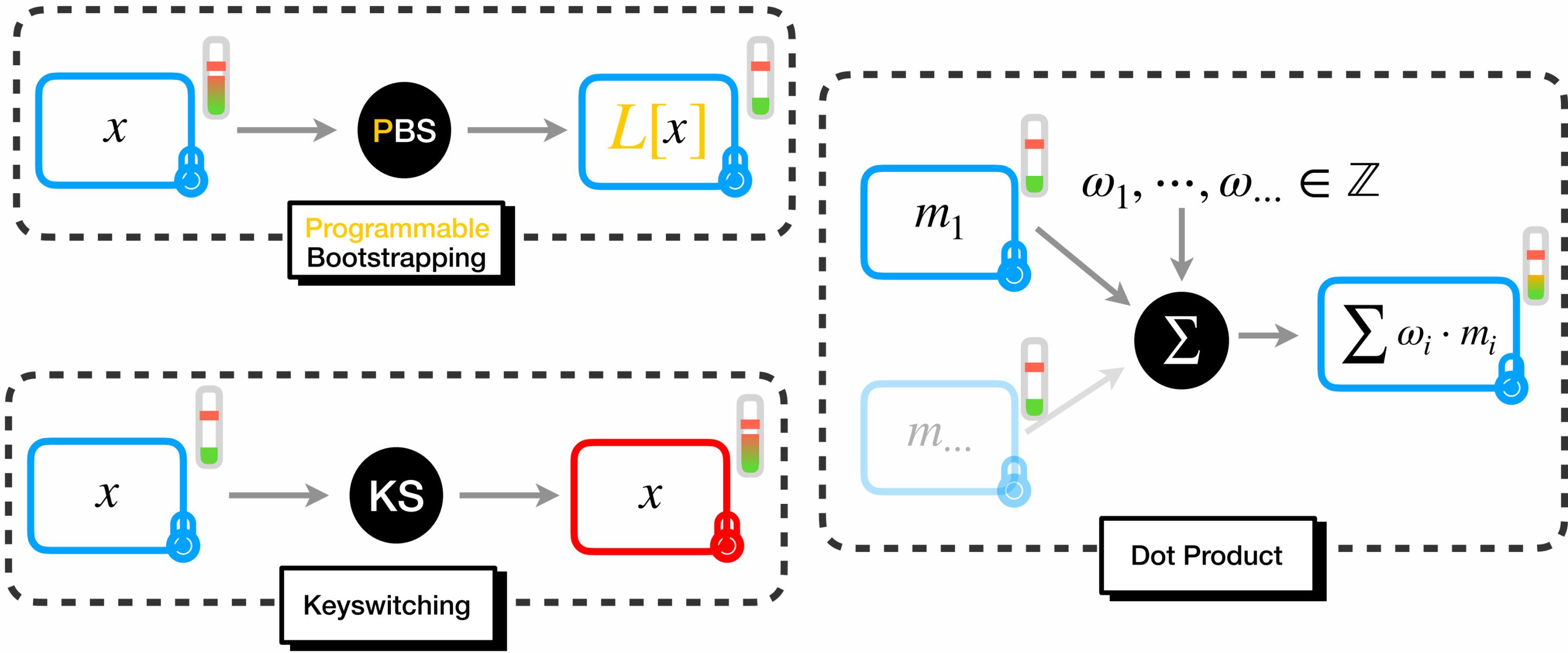
# FHE



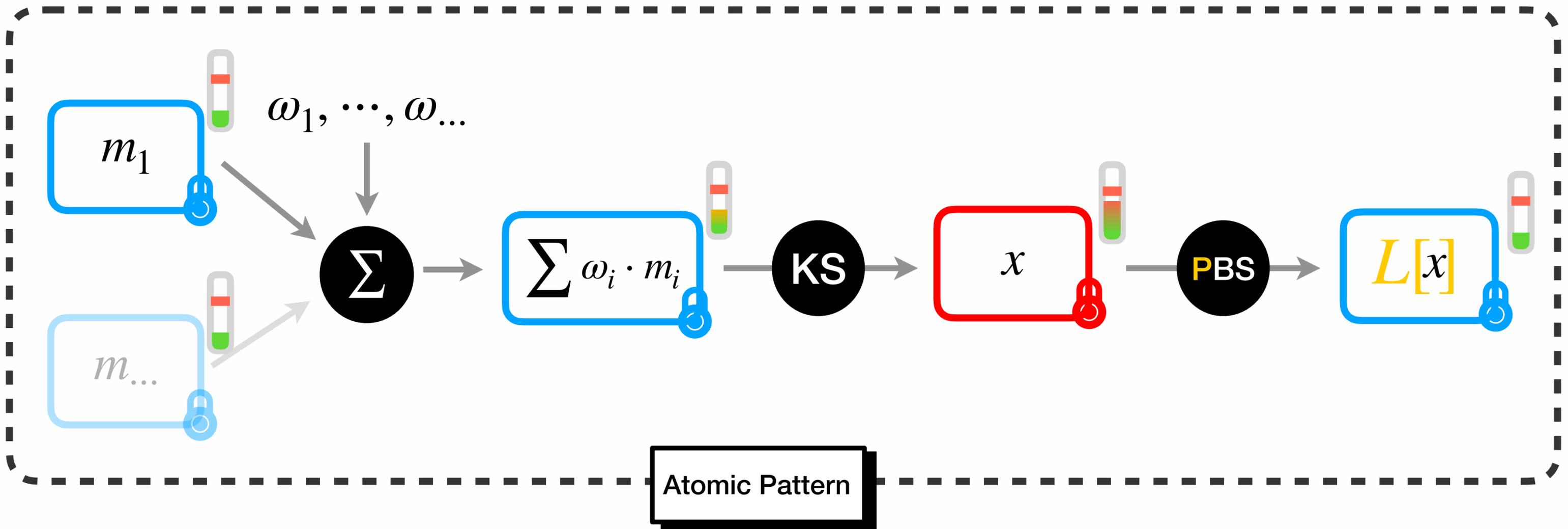
# TFHE



# TFHE building blocks



# Optimal arrangement of TFHE building blocks



# Enhancing TFHE

## Boolean

Classical approach with TFHE

Independent steps: translation, boolean circuit optimization, parametrization

Leverage known techniques for boolean circuits

Support wide range of use cases

Support large precision

Fixed set of parameters

## Integer

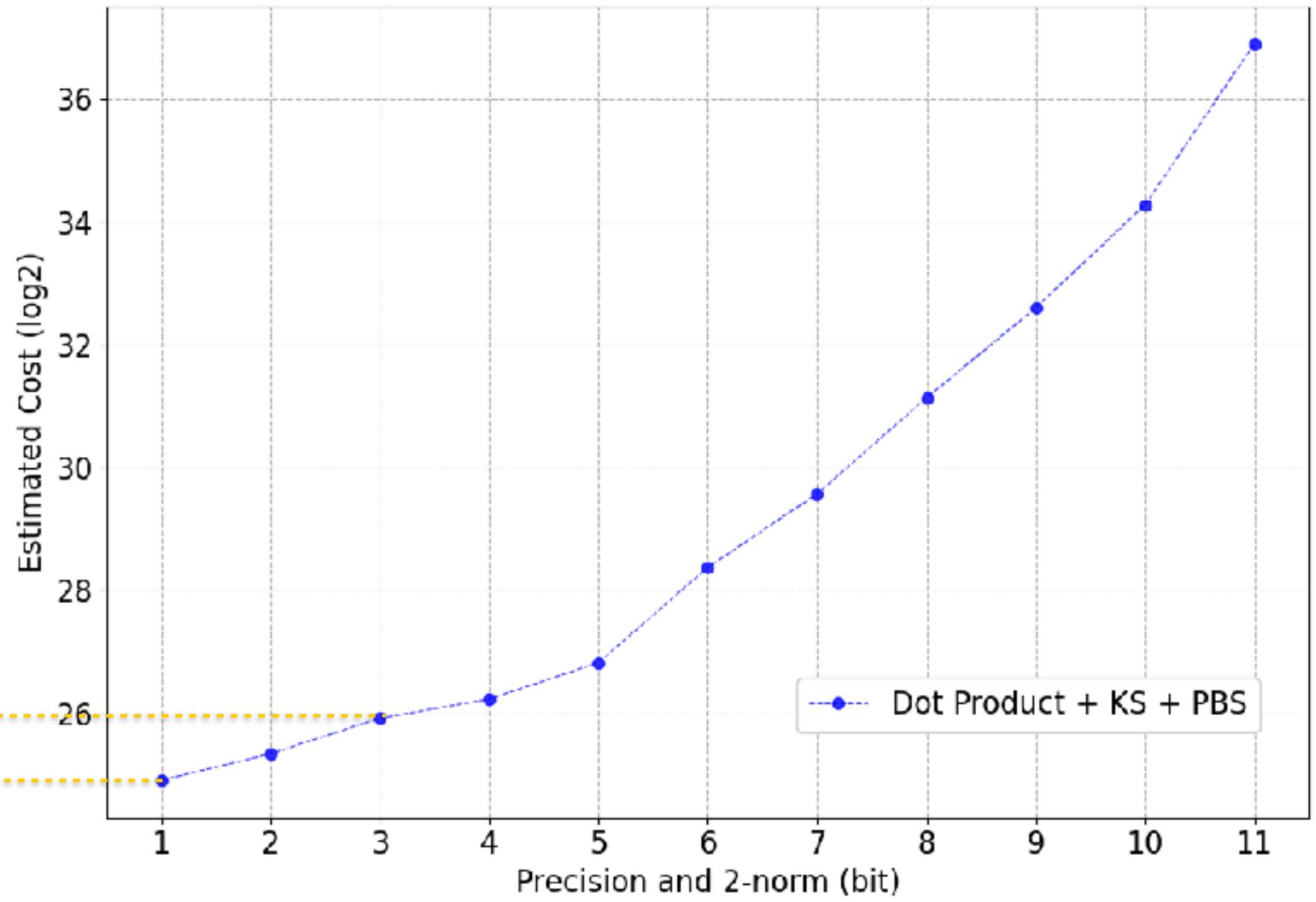
PBS-free leveled operations (additions, subtractions, ...)

Generalization of the boolean approach

Perfect for small integer use-cases

Approximate paradigm

# Enhancing TFHE



11.7 ms/bit  $\Leftarrow$  35.1 ms  
 28.9 ms/bit  $\Leftarrow$  28.9 ms

# Enhancing TFHE



## New paradigms

Short integers (1-10 bits)  
instead of boolean-only

One or several parameter set  
for a given DAG

Failure probability at DAG level



## New algorithms

WoP-PBS

Rounded-PBS



## Needs

Optimal translation of a plain  
DAG into a TFHE DAG

Pick parameters

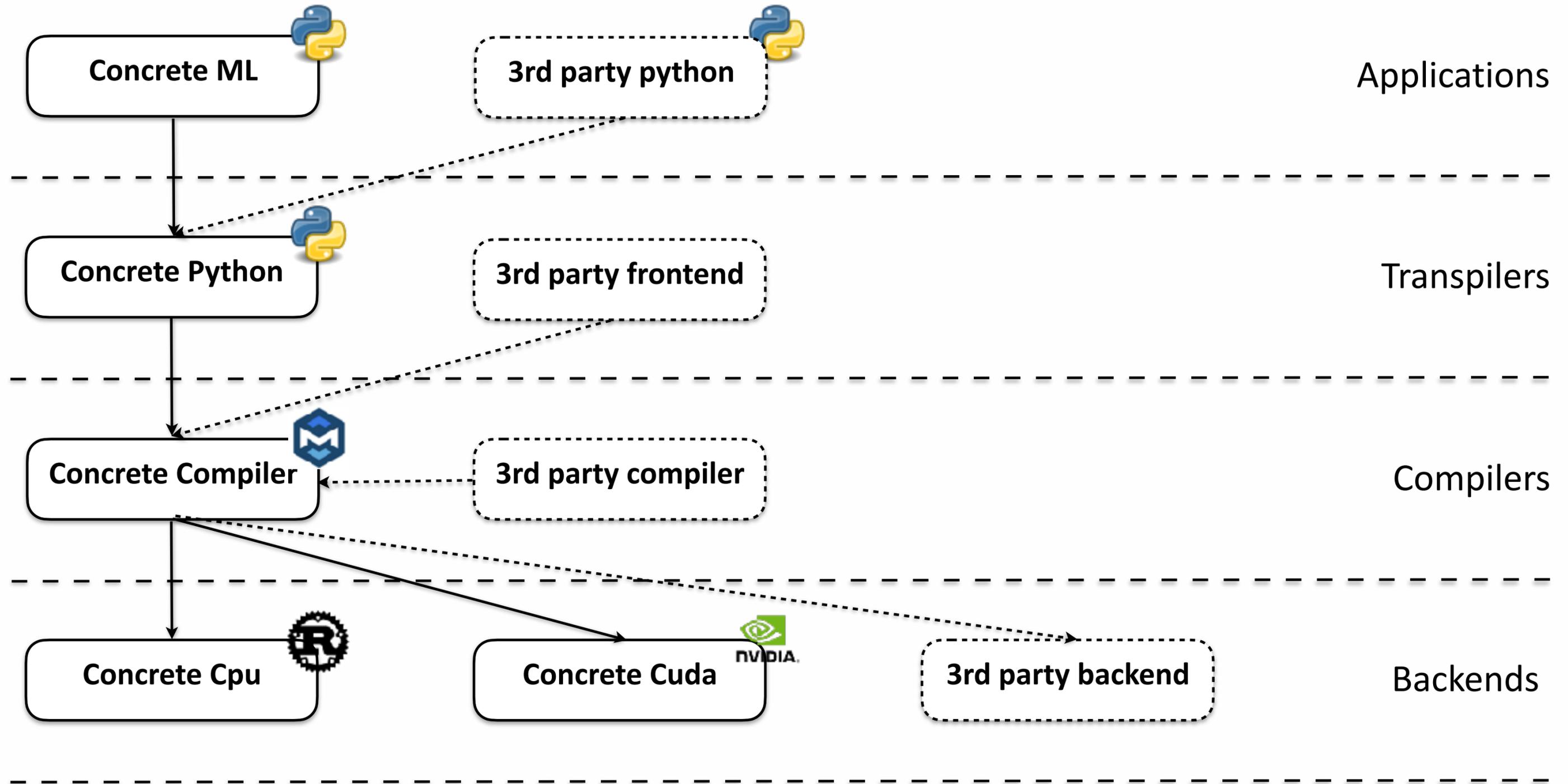
Easy-to-use toolchain

Target non-crypto users

# Concrete

A modular framework for FHE applications

# Concrete: a modular framework



# Concrete Python

Transpiler for Concrete Compiler

# Concrete Python

## An easy to use frontend

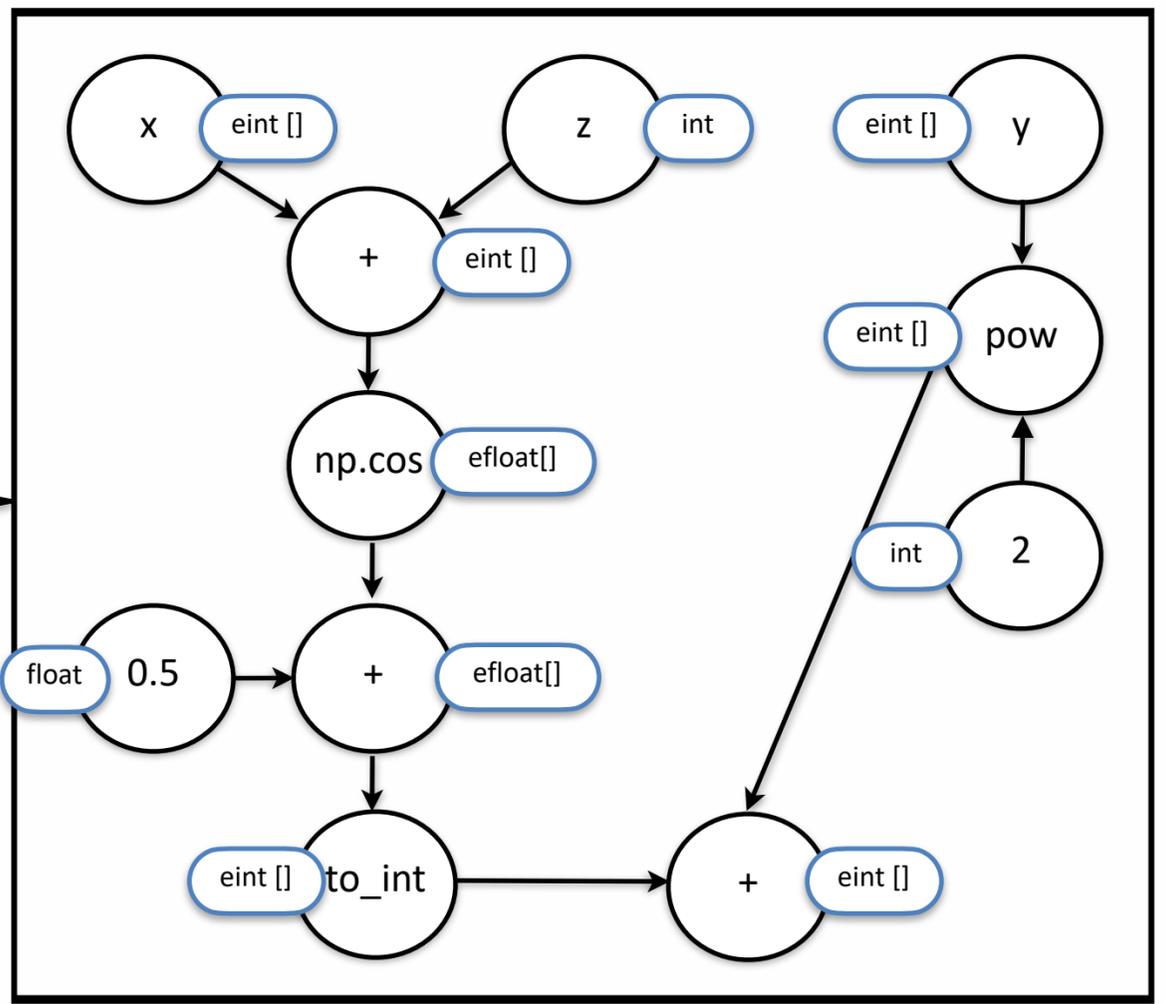
- Simple interface to **compile python programs**
- **Type inference** based on the dataset evaluation
- **Client and server API** to run FHE evaluation
- Seamless conversion of univariate **floating point and integer function** to table lookup
- **Extensive support of python** and numpy standard functions on scalar and tensor values

```
1  from concrete import fhe
2
3  import numpy as np
4
5  # Define your standard python function
6  @fhe.compiler({"x": "encrypted", "y": "encrypted"})
7  def f(x, y):
8      |   return (x + y) ** 2
9
10 # Compile with an input set
11 circuit = f.compile([(0, 2), (3, 4)], verbose=True)
12
13 # Encrypt data and export public evaluation material
14 encrypted_args = circuit.client.encrypt(1, 2)
15 eval_keys = circuit.client.evaluation_keys
16
17 # Evaluate on encrypted data
18 public_res = circuit.server.run(encrypted_args, eval_keys)
19
20 # Decrypt and assert that is equal to the clear evaluation
21 assert(f(1,2) == circuit.client.decrypt(public_res))
```

# Concrete Python: The transpilation pipeline

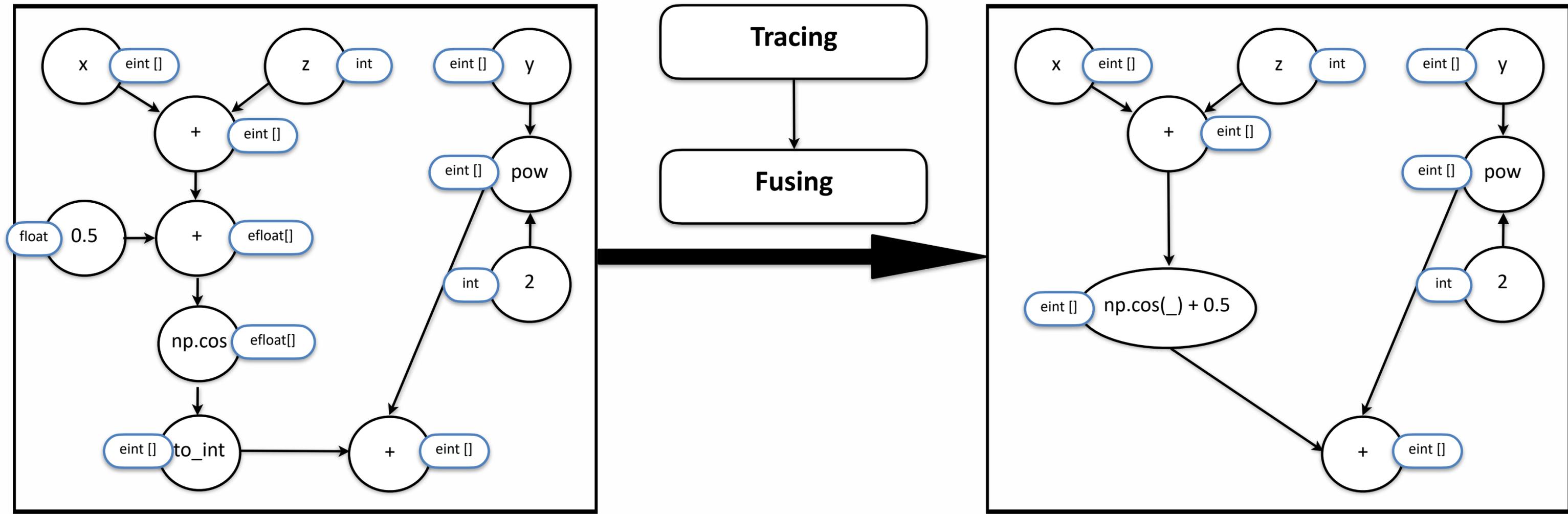
```
1 from concrete import fhe
2
3 import numpy as np
4
5 @fhe.compiler({"x": "encrypted",
6              "y": "encrypted",
7              "z": "clear"})
8 def f(x, y, z):
9     x = x + z
10    x = np.cos(x)
11    x = (x + 0.5).astype(np.int64)
12    y = y**2
13    return x + y
14
15 circuit = f.compile([([0,3], [2,4], 12)])
```

Tracing



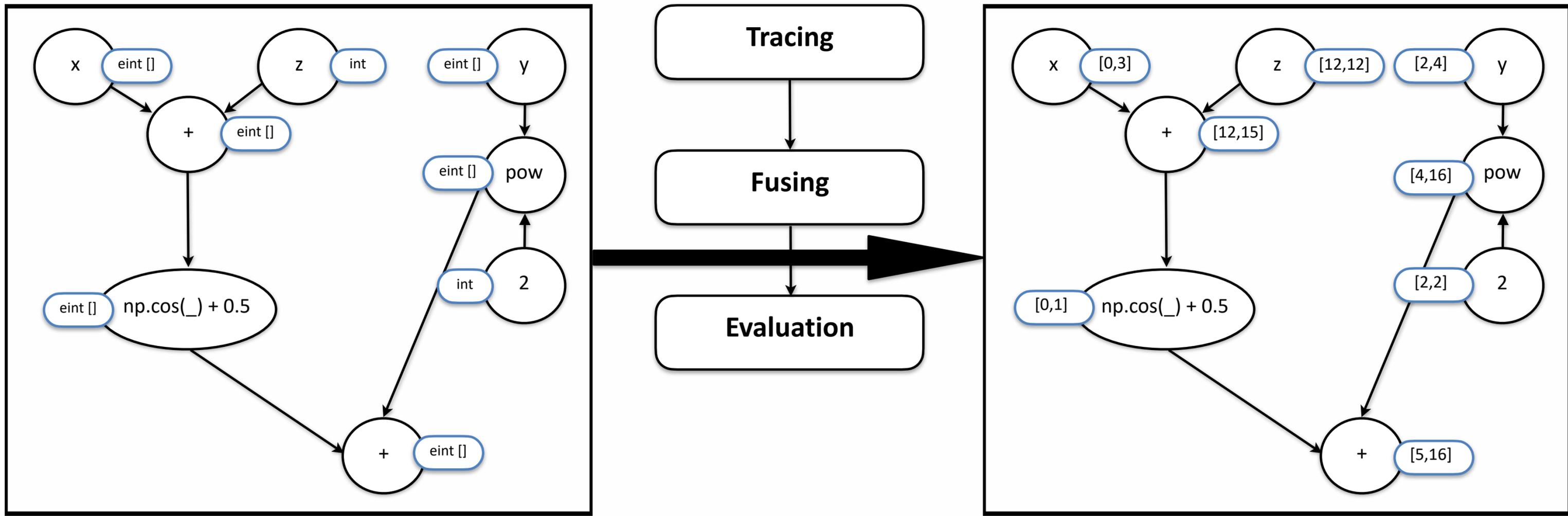
Trace the execution of the **python function** to build a **computation dag**

# Concrete Python: The transpilation pipeline



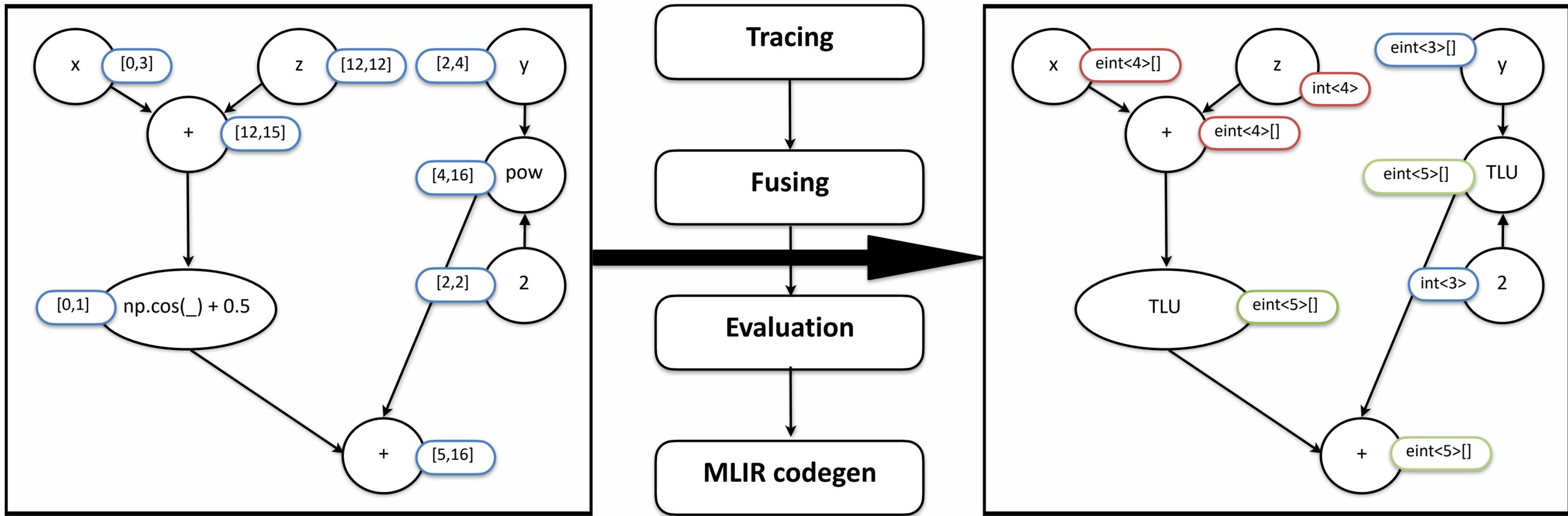
Fuse **floating** point subgraph **to** an **integer** node

# Concrete Python: The transpilation pipeline



Evaluate the computation dag with the dataset to **compute nodes bounds**

# Concrete Python: The transpilation pipeline



**Assign bitwidth** to connected TLU free subgraph and **generate FHE MLIR code**

# Concrete Python: MLIR generated code

```

1  func.func @main(%arg0: tensor<2x!FHE.eint<4>>, %arg1: tensor<2x!FHE.eint<3>>, %arg2: i5) -> tensor<2x!FHE.eint<5>> {
2      // Boiler plate code to transform scalar integer to one element tensor
3      %from_elements = tensor.from_elements %arg2 : tensor<1xi5>
4
5      // x + z
6      %0 = "FHELinalg.add_eint_int"(%arg0, %from_elements)
7      |   : (tensor<2x!FHE.eint<4>>, tensor<1xi5>) -> tensor<2x!FHE.eint<4>>
8
9      // np.cos(_) + 0.5
10     %cst = arith.constant dense<[1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0]> : tensor<16xi64>
11     %1 = "FHELinalg.apply_lookup_table"(%0, %cst)
12     |   : (tensor<2x!FHE.eint<4>>, tensor<16xi64>) -> tensor<2x!FHE.eint<5>>
13
14     // pow(_, 2)
15     %cst_0 = arith.constant dense<[0, 1, 4, 9, 16, 25, 36, 49]> : tensor<8xi64>
16     %2 = "FHELinalg.apply_lookup_table"(%arg1, %cst_0)
17     |   : (tensor<2x!FHE.eint<3>>, tensor<8xi64>) -> tensor<2x!FHE.eint<5>>
18
19     // _ + _
20     %3 = "FHELinalg.add_eint"(%1, %2)
21     |   : (tensor<2x!FHE.eint<5>>, tensor<2x!FHE.eint<5>>) -> tensor<2x!FHE.eint<5>>
22     return %3 : tensor<2x!FHE.eint<5>>
23 }

```

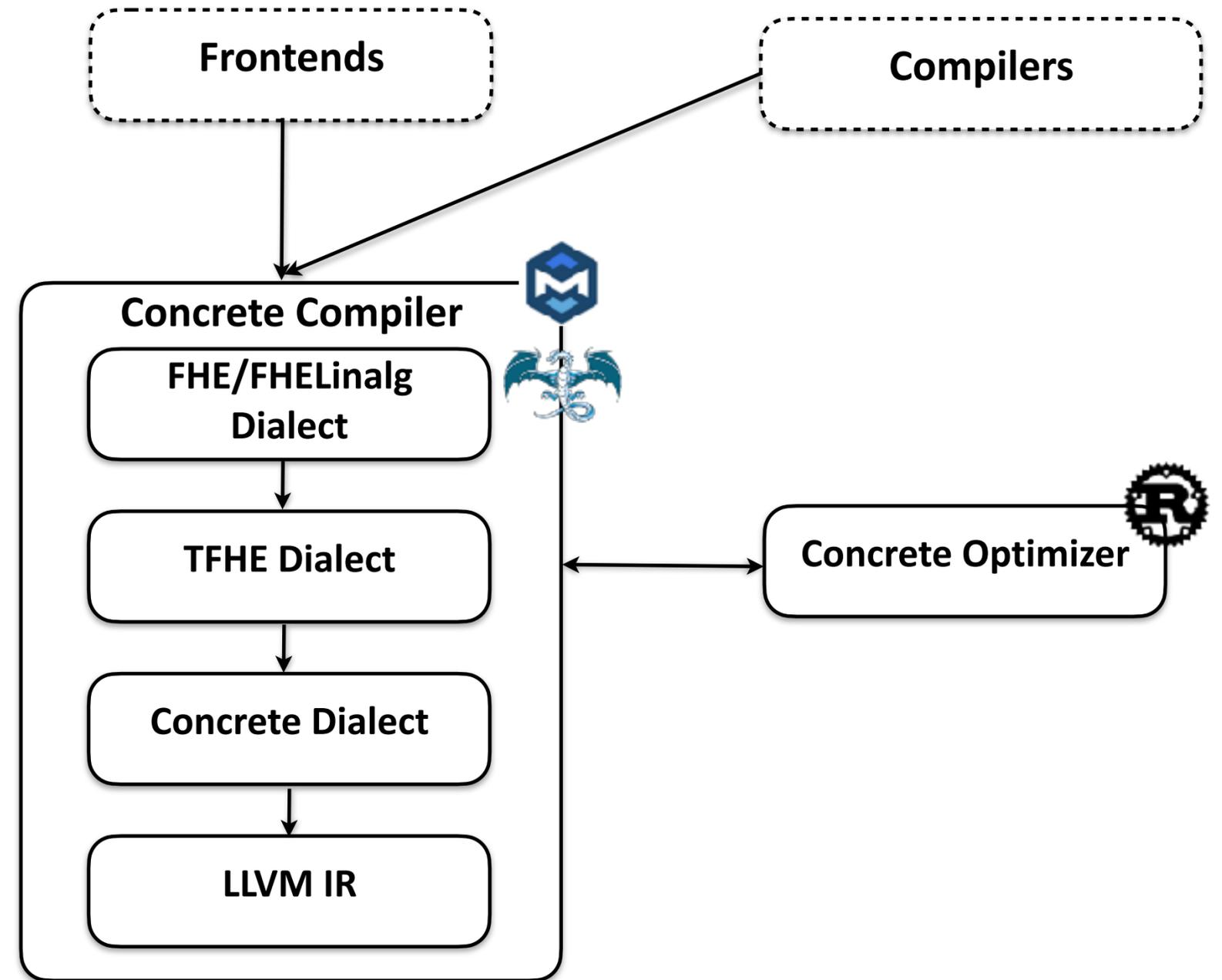
# Concrete Compiler

From crypto-free representation to TFHE executable

# Concrete Compiler

## High-Level Overview (1)

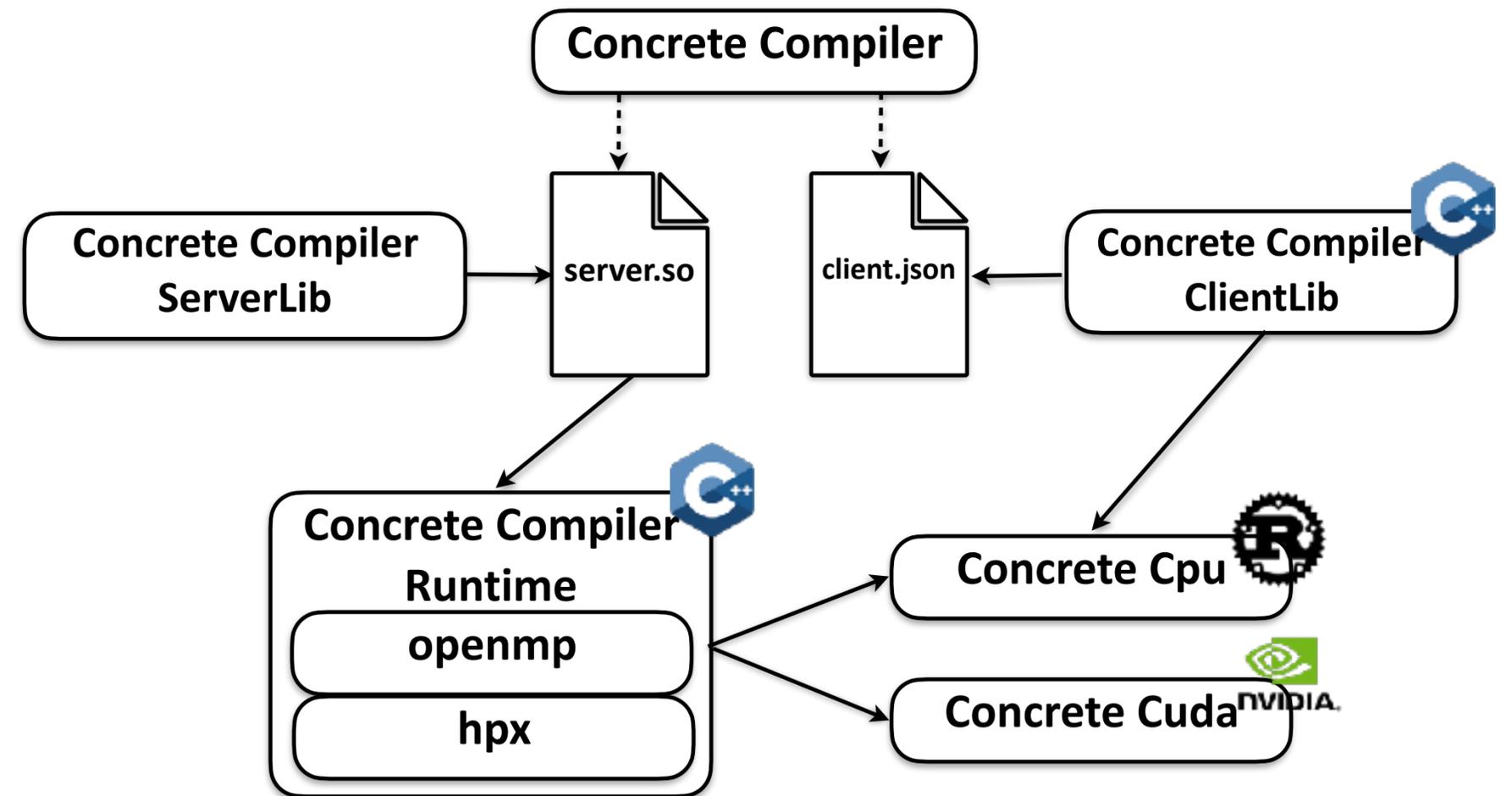
- **MLIR**-based compiler to be reusable and leverage community effort on common problems
- **Concrete Optimizer** to solve TFHE parametrization problems
- **LLVM Toolchain** to produce binary library



# Concrete Compiler

## High-Level Overview (2)

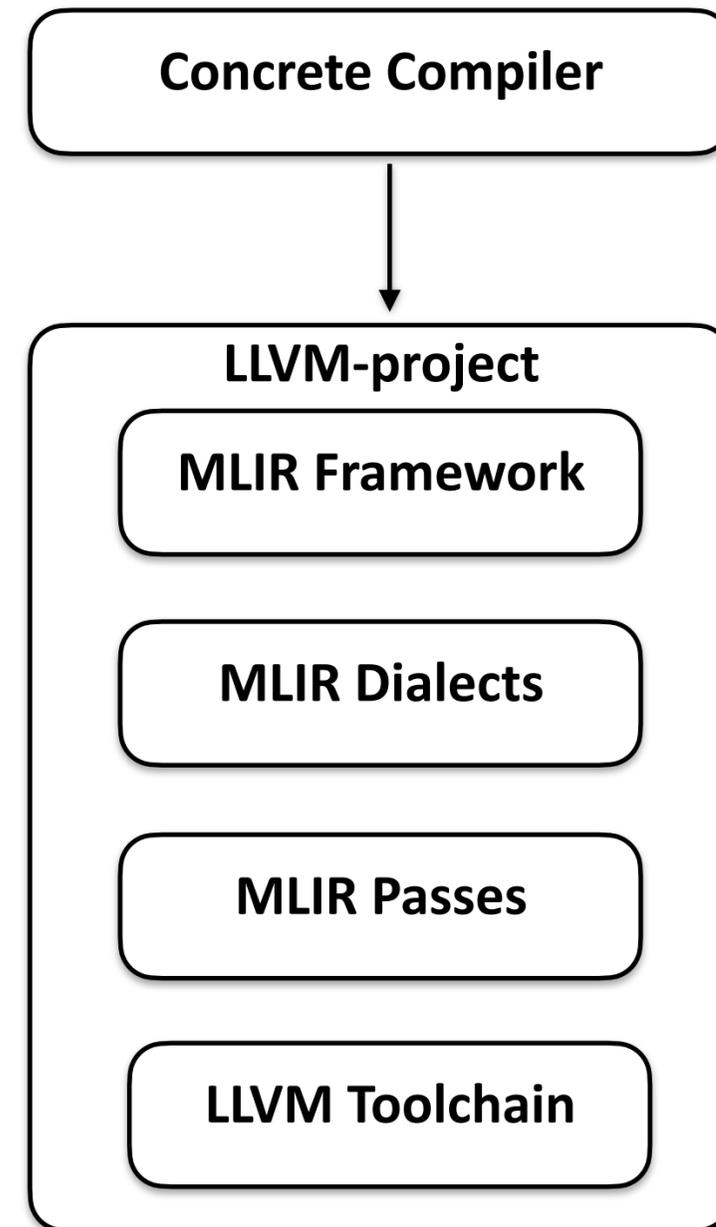
- Runtime linked with **OpenMP** and **HPX** libraries for loop parallelism and task scheduling
- Runtime linked to **Concrete CPU/GPU** backends to use the fastest hand optimized TFHE implementation
- **Client and Server toolkit** to use compilation artifacts



# Concrete Compiler

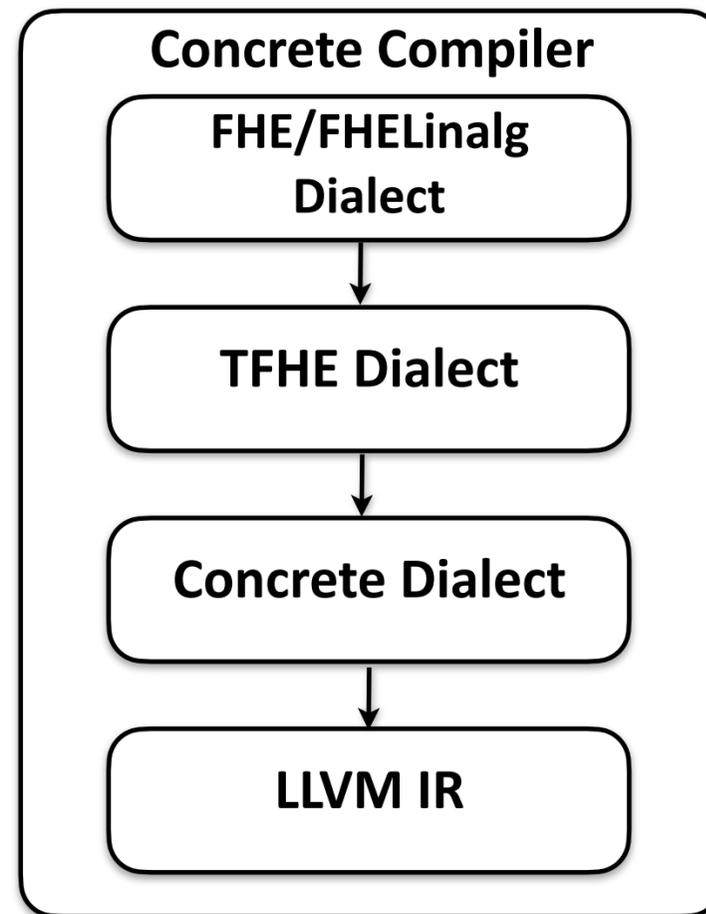
## Why MLIR?

- **MLIR infrastructure** allows to **reduce the cost** of building Concrete Compiler
- **Standard MLIR Dialects** to **model common** compiler **abstraction** (tensor, memref, linalg, scf, .omp, ..)
- **Standard MLIR Passes** to **solve common** compiler **problems** (canonicalization, linalg generalization, dead code elimination, bufferization...)
- Leverage **LLVM toolchain** to produce efficient binaries
- Allows for a **reusable definition of FHE-specific dialect** and optimization passes



# Concrete Compiler

## Specific Dialects



```

1  %0 = "FHE.mul_eint"(%arg0, %arg1):
2  |  (!FHE.eint<2>, !FHE.eint<2>) -> (!FHE.eint<2>)
  
```

**FHE Dialect** defines crypto-free FHE **types** and **scalar operators**

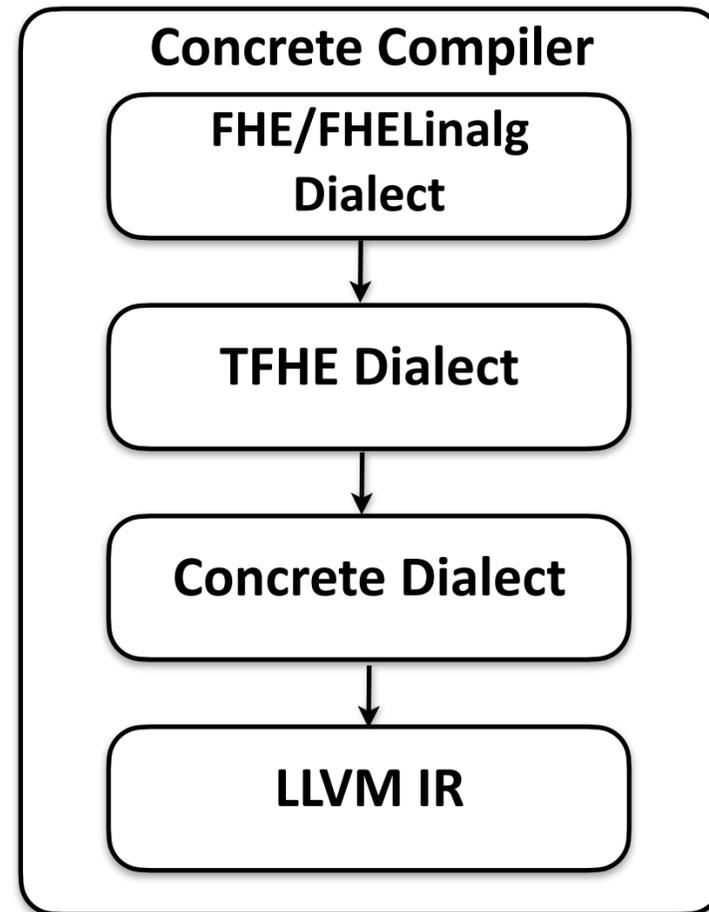
```

1  %0 = "FHELinalg.matmul_eint_int"(%x, %y):
2  |  (tensor<4x3x!FHE.eint<2>>, tensor<3x2xi3>) -> tensor<4x2x!FHE.eint<2>>
  
```

**FHELinalg Dialect** defines very high level **tensor operators**

# Concrete Compiler

## Specific Dialects



```

1  %2 = "TFHE.keyswitch_glwe"(%0) {key = #TFHE.ksk<sk<0,1,1280>, sk<1,1,677>, 3, 4>}
2  | : (!TFHE.glwe<sk<0,1,1280>>) -> !TFHE.glwe<sk<1,1,677>>
3  %3 = "TFHE.bootstrap_glwe"(%2, %1) {key = #TFHE.bsk<sk<1,1,677>, sk<0,1,1280>, 256, 5, 1, 15>}
4  | : (!TFHE.glwe<sk<1,1,677>>, tensor<256xi64>) -> !TFHE.glwe<sk<0,1,1280>>
  
```

**TFHE Dialect** introduces crypto-system dependent parameters and operators

```

1  %0 = "Concrete.add_lwe_tensor"(%arg0, %arg1)
2  | : (tensor<1281xi64>, tensor<1281xi64>) -> tensor<1281xi64>
  
```

**Concrete Dialect** represents unabstracted implementation operators to prepare the codegen

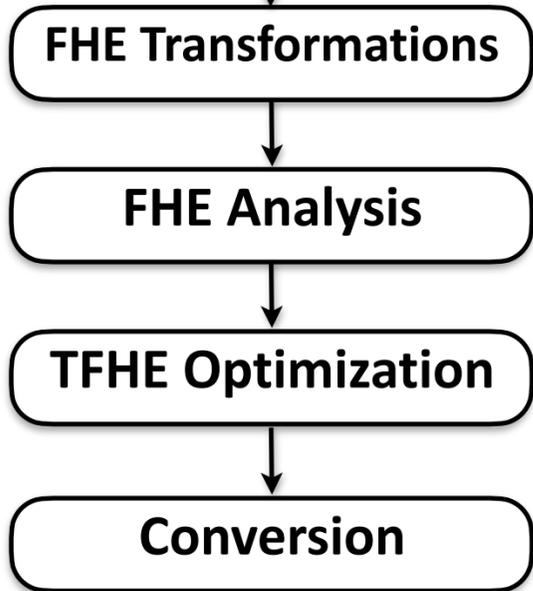
# Concrete Compiler

## High Level Pipeline

- A set of **transformations** passes to translate non natively supported TFHE operators
- A set of **analysis** passes to **build** the FHE **constraint DAG**
- A set of **conversion** passes to go from **FHE/FHELinalg** dialects to TFHE following Concrete Optimizer re-writing guidelines

```

1  %0 = "FHE.mul_eint"(%arg0, %arg1):
2  |  (IFHE.eint<2>, IFHE.eint<2>) -> (IFHE.eint<2>)
    
```



```

1  %c4611686018427387904_i64 = arith.constant 4611686018427387904 : i64
2  %cst = arith.constant dense<[0, 0, 1, 0]> : tensor<4xi64>
3  %cst_0 = arith.constant dense<[0, 0, 1, 2]> : tensor<4xi64>
4  %0 = "TFHE.add_glwe"(%arg0, %arg1) : (!TFHE.glwe<sk<0,1,1280>>, !TFHE.glwe<sk<0,1,1280>>)
5  %1 = "TFHE.encode_expand_lut_for_bootstrap"(%cst_0) {isSigned = false, outputBits = 2 : i32} : !TFHE.glwe<sk<0,1,1280>>
6  %2 = "TFHE.keyswitch_glwe"(%0) {key = #TFHE.ksk<sk<0,1,1280>, sk<1,1,677>, 3, 4>} : (!TFHE.glwe<sk<0,1,1280>>, !TFHE.glwe<sk<1,1,677>>)
7  %3 = "TFHE.bootstrap_glwe"(%2, %1) {key = #TFHE.bsk<sk<1,1,677>, sk<0,1,1280>, 256, 5, 1, 1>} : (!TFHE.glwe<sk<0,1,1280>>, !TFHE.glwe<sk<1,1,677>>, !TFHE.glwe<sk<0,1,1280>>)
8  %4 = "TFHE.neg_glwe"(%arg1) : (!TFHE.glwe<sk<0,1,1280>>) -> !TFHE.glwe<sk<0,1,1280>>
9  %5 = "TFHE.add_glwe"(%arg0, %4) : (!TFHE.glwe<sk<0,1,1280>>, !TFHE.glwe<sk<0,1,1280>>) -> !TFHE.glwe<sk<0,1,1280>>
10 %6 = "TFHE.encode_expand_lut_for_bootstrap"(%cst) {isSigned = true, outputBits = 2 : i32} : !TFHE.glwe<sk<0,1,1280>>
11 %7 = "TFHE.add_glwe_int"(%5, %c4611686018427387904_i64) : (!TFHE.glwe<sk<0,1,1280>>, i64)
12 %8 = "TFHE.keyswitch_glwe"(%7) {key = #TFHE.ksk<sk<0,1,1280>, sk<1,1,677>, 3, 4>} : (!TFHE.glwe<sk<0,1,1280>>, !TFHE.glwe<sk<1,1,677>>)
13 %9 = "TFHE.bootstrap_glwe"(%8, %6) {key = #TFHE.bsk<sk<1,1,677>, sk<0,1,1280>, 256, 5, 1, 1>} : (!TFHE.glwe<sk<0,1,1280>>, !TFHE.glwe<sk<1,1,677>>, !TFHE.glwe<sk<0,1,1280>>)
14 %10 = "TFHE.neg_glwe"(%9) : (!TFHE.glwe<sk<0,1,1280>>) -> !TFHE.glwe<sk<0,1,1280>>
15 %11 = "TFHE.add_glwe"(%3, %10) : (!TFHE.glwe<sk<0,1,1280>>, !TFHE.glwe<sk<0,1,1280>>) -> !TFHE.glwe<sk<0,1,1280>>
    
```

# Concrete Compiler

## Automatic loop parallelism

High-level operators are lowered to `linalg.generic` with parallel iterators

```
1 %0 = "FHELinalg.apply_lookup_table"(%arg0, %arg1)
2 | : (tensor<2x3x4x!FHE.eint<2>>, tensor<4xi64>) -> tensor<2x3x4x!FHE.eint<2>>
```

```
1 %1 = linalg.generic {iterator_types = ["parallel", "parallel", "parallel"], ...} ... {
2 | ...
3 | %2 = "FHE.apply_lookup_table"(%in, %arg1) : (!FHE.eint<2>, tensor<4xi64>) -> !FHE.eint<2>
4 | linalg.yield %2 : !FHE.eint<2>
5 } -> tensor<2x3x4x!FHE.eint<2>>
```

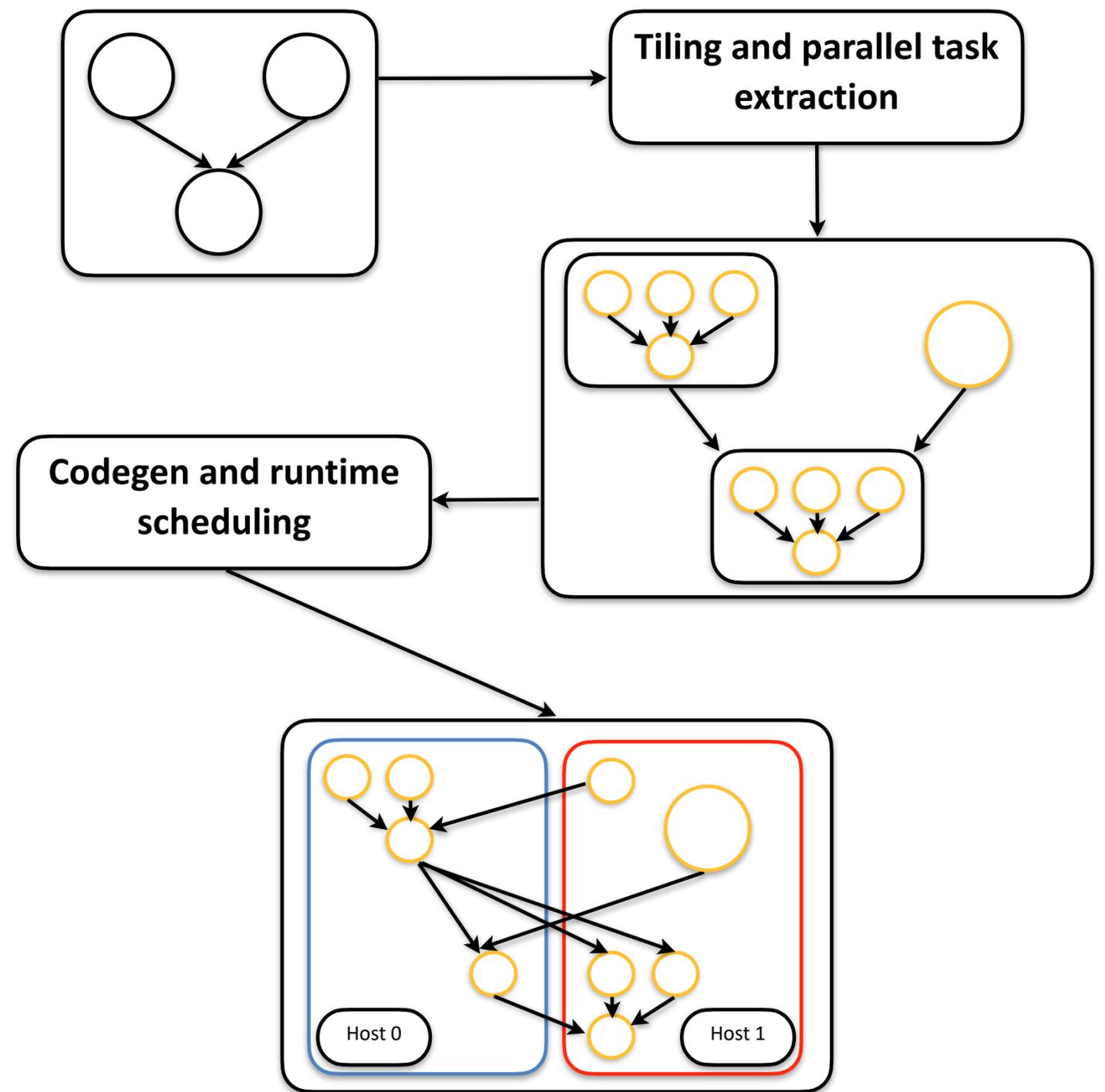
```
1 ...
2 omp_loop.body: ; preds = %omp_loop.cond
3 | %9 = add i64 %omp_loop.iv, %5
4 | %10 = mul i64 %9, 1
5 | %11 = add i64 %10, 0
6 | br label %omp_wsloop.region
7 |
8 omp_wsloop.region: ; preds = %omp_loop.body
9 | %12 = call ptr @llvm.stacksave()
10 | br label %omp_wsloop.region3
11 |
12 omp_wsloop.region3: ; preds = %omp_wsloop.region
13 | %13 = srem i64 %11, 4
14 | %14 = sdiv i64 %11, 4
15 | ...
```

Rely on the existing MLIR infrastructure to generate `llvm-ir` with OpenMP annotations

# Concrete Compiler

## Dataflow task parallelism

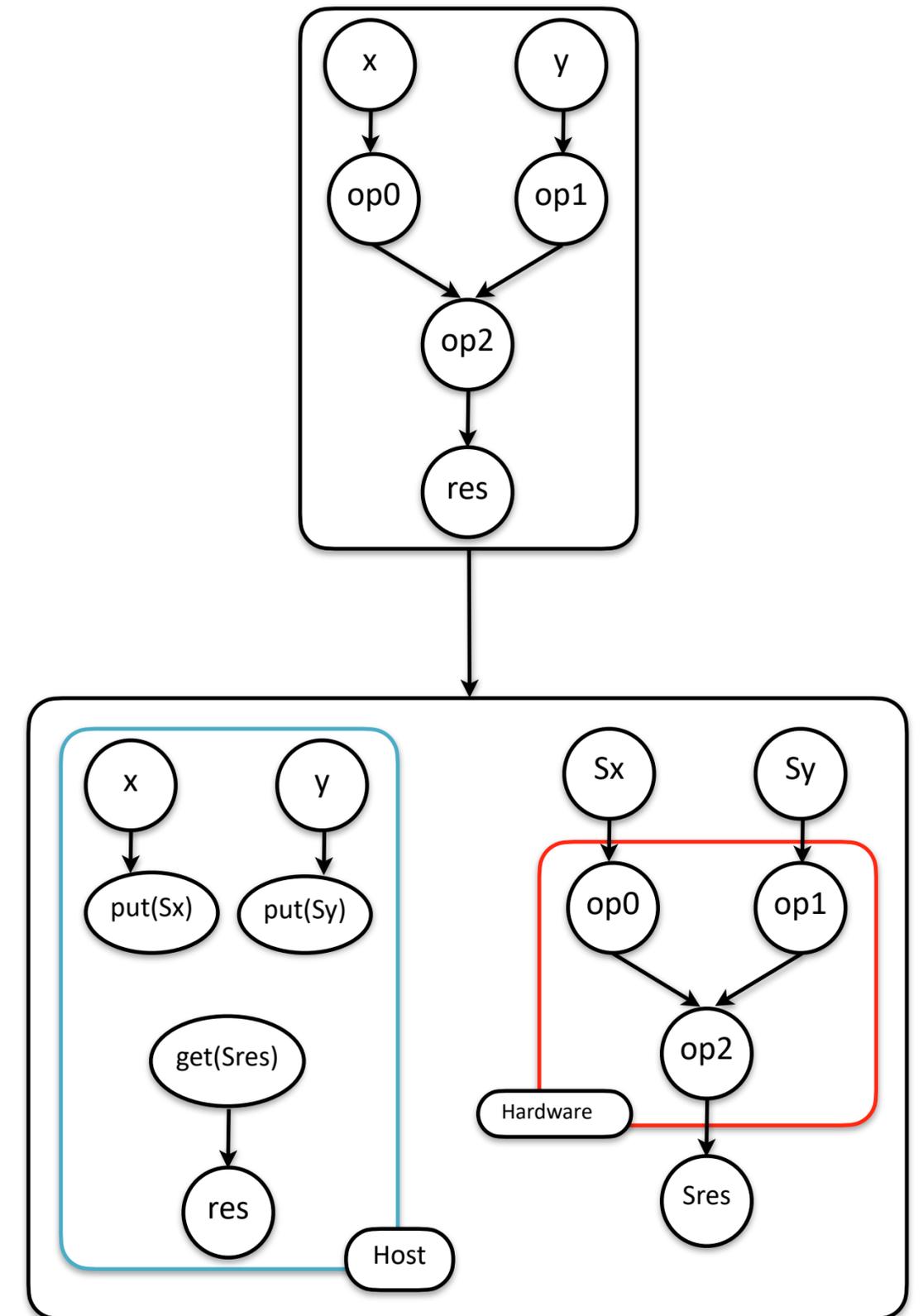
- Motivation: extract **parallel tasks** with dataflow analysis where loop parallelism is not available and execute tasks on **distributed systems**
- A dialect to express **high-level dataflow tasks** and **runtime abstractions**
- A set of passes **from building a dataflow task graph to generating code** for the dataflow runtime
- Reusing MLIR **tiling** infrastructure to expose **more parallelism** and control **granularity**
- A **HPX-based runtime** to schedule **dataflow** tasks within and across hosts



# Concrete Compiler

## SDFG: Static DataFlow Graph

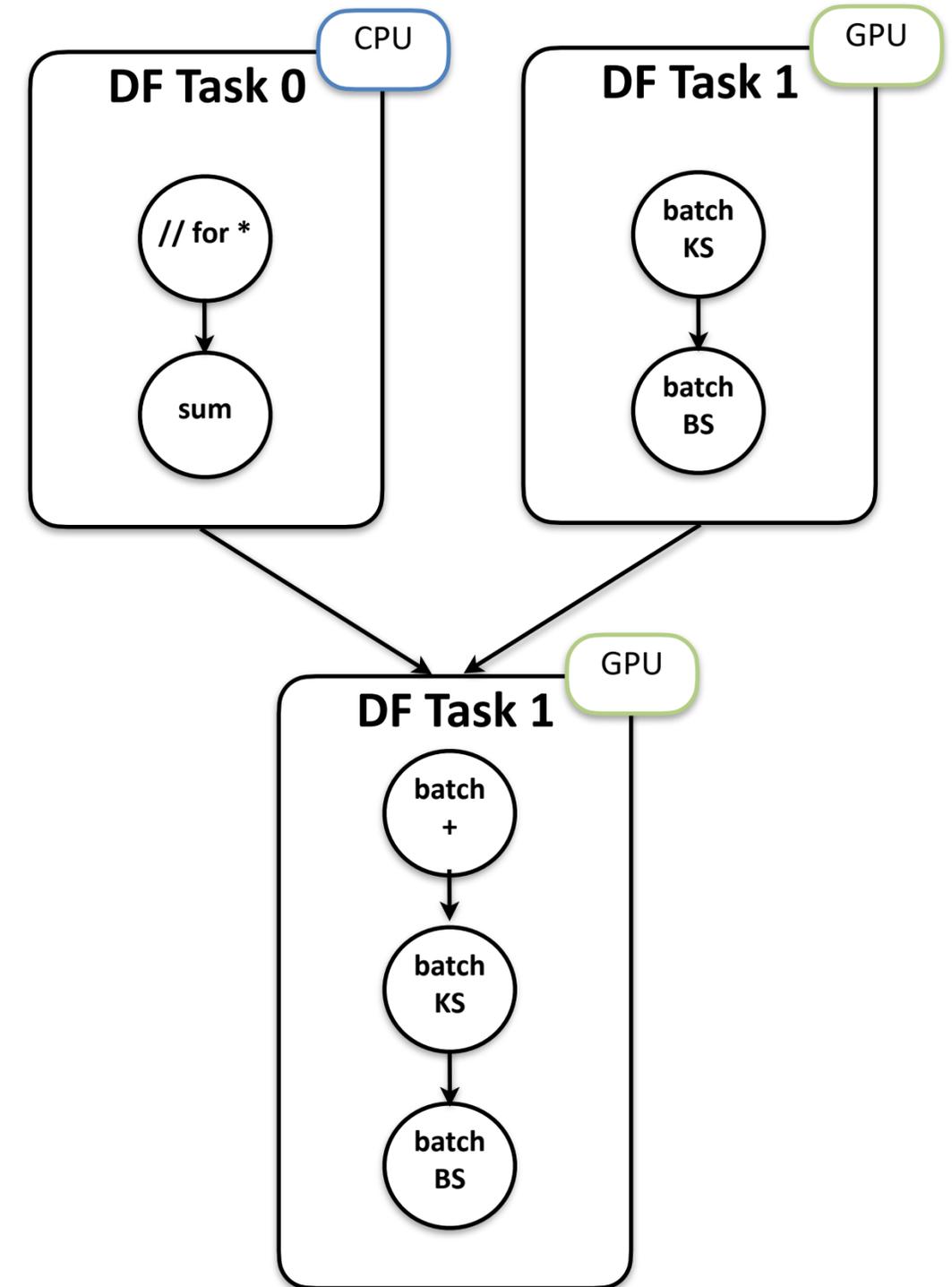
- Motivation: Efficiently **offload** a subgraph of tasks **on hardware accelerators** and maximizing data reuse on device (minimize back and forth data transfer from host to device)
- A dialect to express static dataflow graphs
- A GPU SDFG based runtime that schedules tasks on multi-GPU hosts



# Concrete Compiler

## Summary parallelism and distribution

- Automatic **loop parallelism** using high level information to lower to **OpenMP**
- Automatic **dataflow parallelism** by generating a dataflow task graph using high level information, and **tiling**
- A **SDFG** Dialect (Static DataFlow Graph) instantiated with high level TFHE primitives to **offload** a whole **TFHE subgraph** (pipeline) for **hardware accelerators** (GPU, ...)
- A **dataflow runtime** based on HPX to implement dataflow tasks' parallelism and **distributed computation**
- A **GPU SDFG runtime** to schedule GPU kernels over multi-GPU hosts



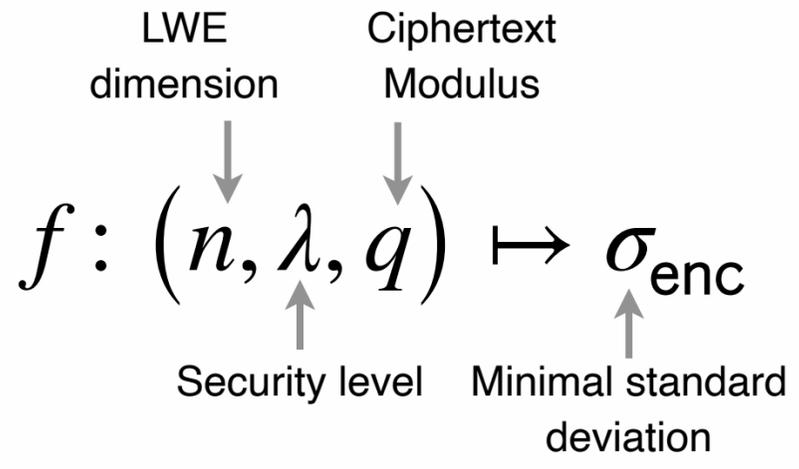
# Concrete Optimizer

An optimizer for TFHE

# Goals



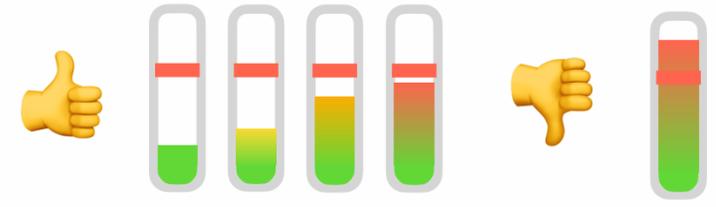
**Security**



→ Using the **lattice estimator**



**Correctness**



→ **Noise Model** to track the noise along the computation



**Efficiency**

→ **Cost Model** as a surrogate of the execution time

# Choice of algorithm

**LUT** LUT/function evaluation  
 $m \in \llbracket 0, 2^p - 1 \rrbracket$



**PBS** TFHE's PBS  
 Very efficient if  $p \in \llbracket 1, 8 \rrbracket$

**WoP - PBS** BBB+'s PBS  
 Very efficient if  $p \in \llbracket 9, 16 \rrbracket$

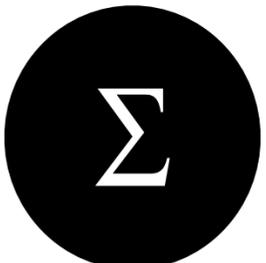
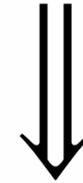
**Rounded PBS** TFHE's PBS on rounded input  
 Very efficient when applicable

[CGGI20] I. Chillotti, N. Gama, M. Georgieva, M. Izabachène. TFHE: Fast Fully Homomorphic Encryption over the Torus. Journal of Cryptology 2020.  
 [BBB+22] L. Bergerat, A. Boudi, Q. Bourgerie, I. Chillotti, D. Ligier, J.-B. Orfila, S. Tap Parameter Optimization & Larger Precision for (T)FHE. [Eprint](#)

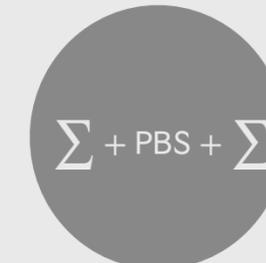
# Choice of algorithm



**Plain Dot Product**  
 $m \in \llbracket 0, 2^p - 1 \rrbracket$

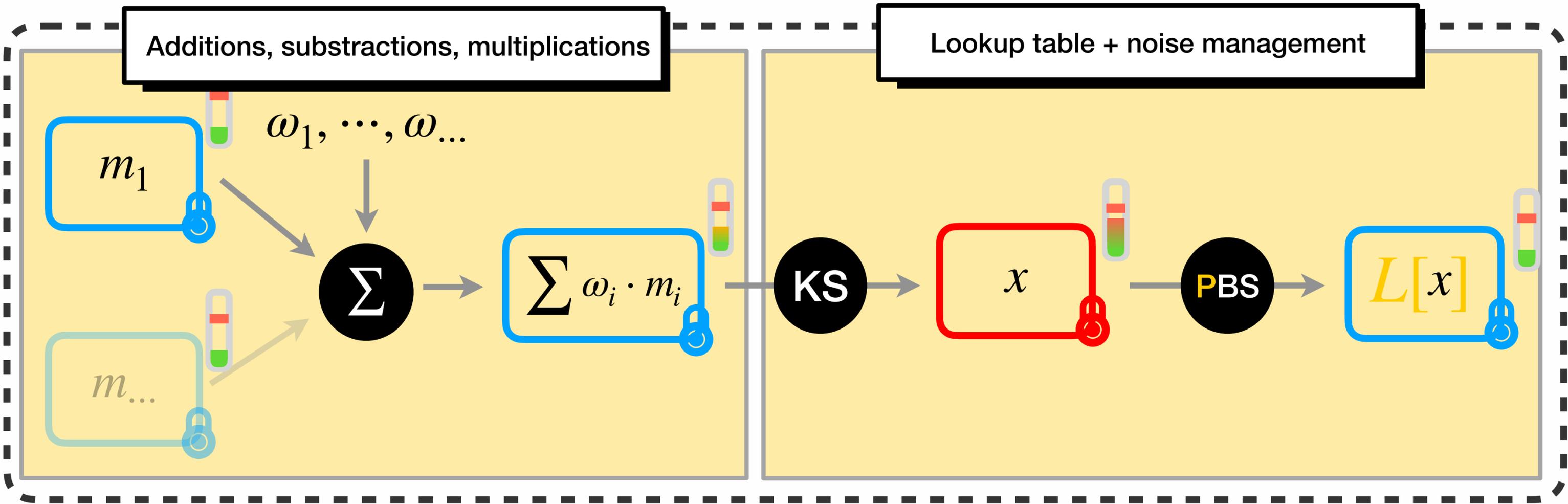


**Dot Product**  
Very efficient  $\nu$  is small

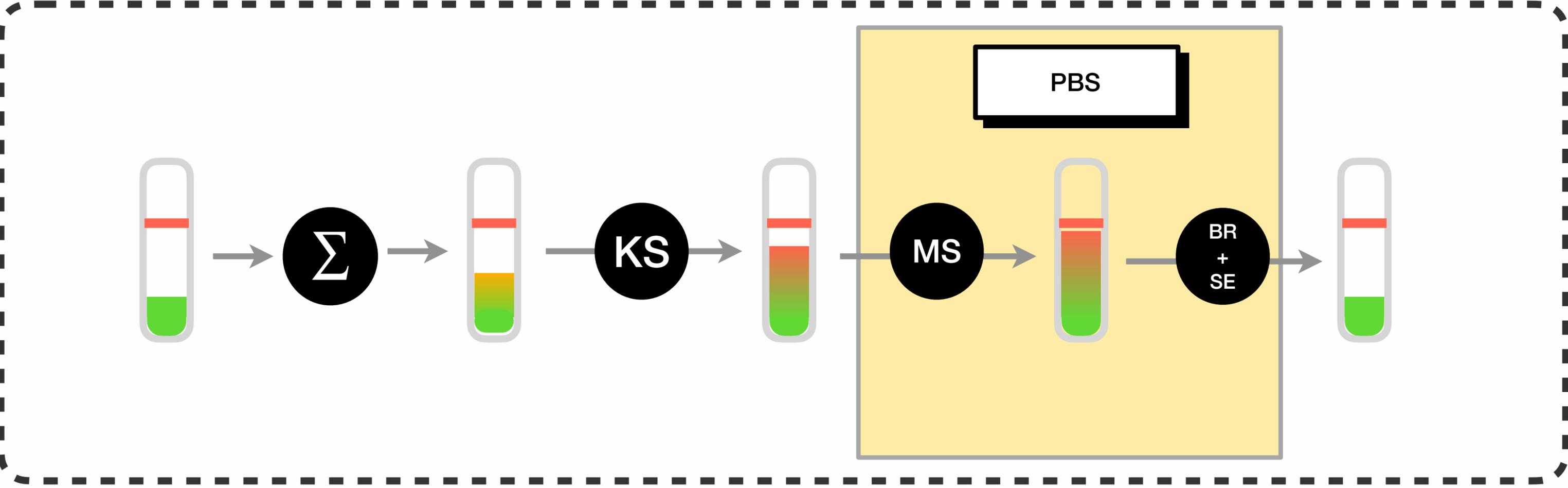


**Dot Product with intra-PBS**  
Very efficient if  $\nu$  is big

# Translation



# Noise analysis of an Atomic Pattern



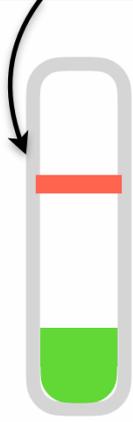
**Noise is increasing between two modulus switching**

# Noise Bound

Noise Bound



is a function of

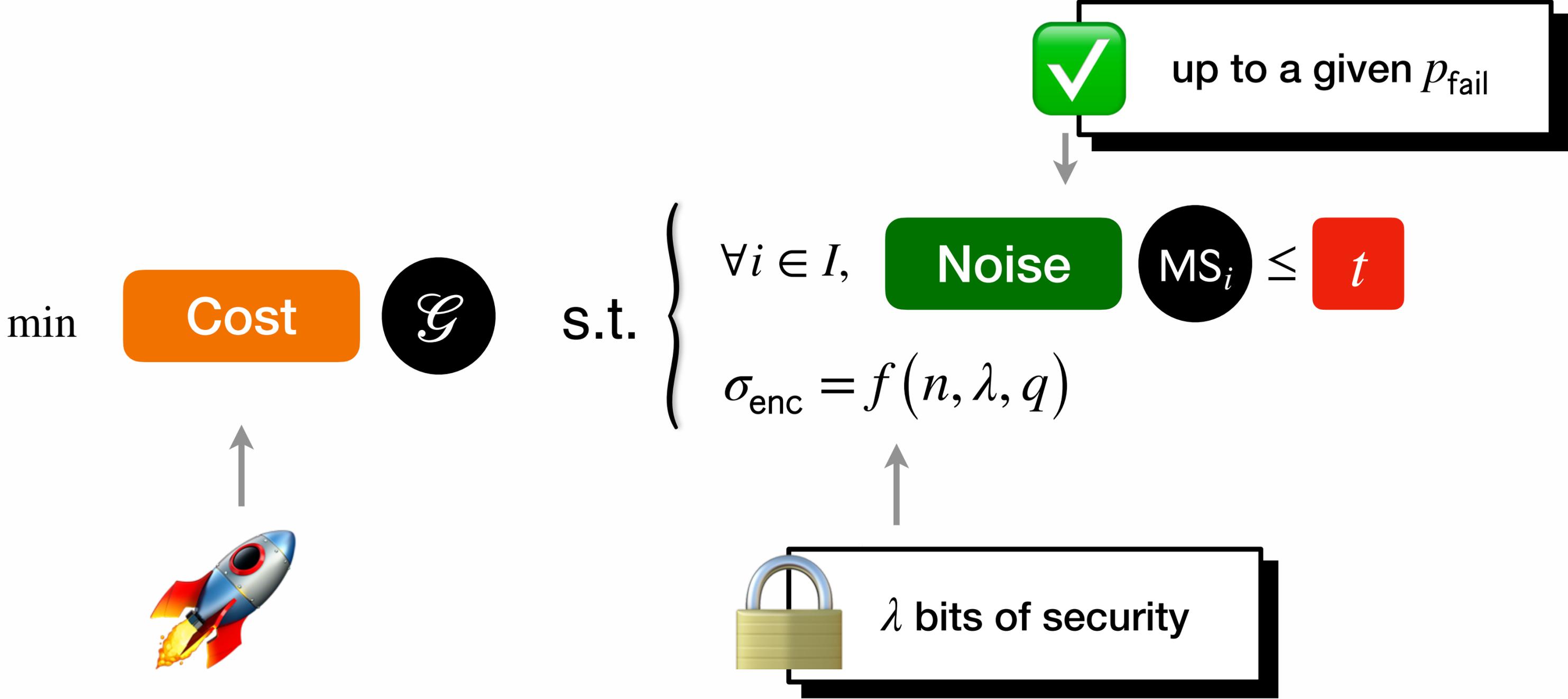


**message precision**

**encoding**  
Natif, CRT, Radix

**failure probability**  
DAG dependent

# Optimization Problem



# Conclusion

# Concrete



## A growing community

821 githubs stars

39 contributors

2916 commits



## A complete stack for FHE

An easy to use frontend

A reusable compiler  
infrastructure

Multi backend integrations



## Built to be fast

TFHE native integer

A specific TFHE optimizer

A compiler pipeline and runtime  
designed to scale

**Thank you.**

**ZAMA**

# Contact and Links

---

[quentin.bourgerie@zama.ai](mailto:quentin.bourgerie@zama.ai)  
[samuel.tap@zama.ai](mailto:samuel.tap@zama.ai)

---

[zama.ai](https://zama.ai)

---

[github.com/zama-ai/concrete](https://github.com/zama-ai/concrete)

---

[community.zama.ai](https://community.zama.ai)