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Damien Ligier, Jean-Baptiste Orfila & **Samuel Tap**

26 March 2023

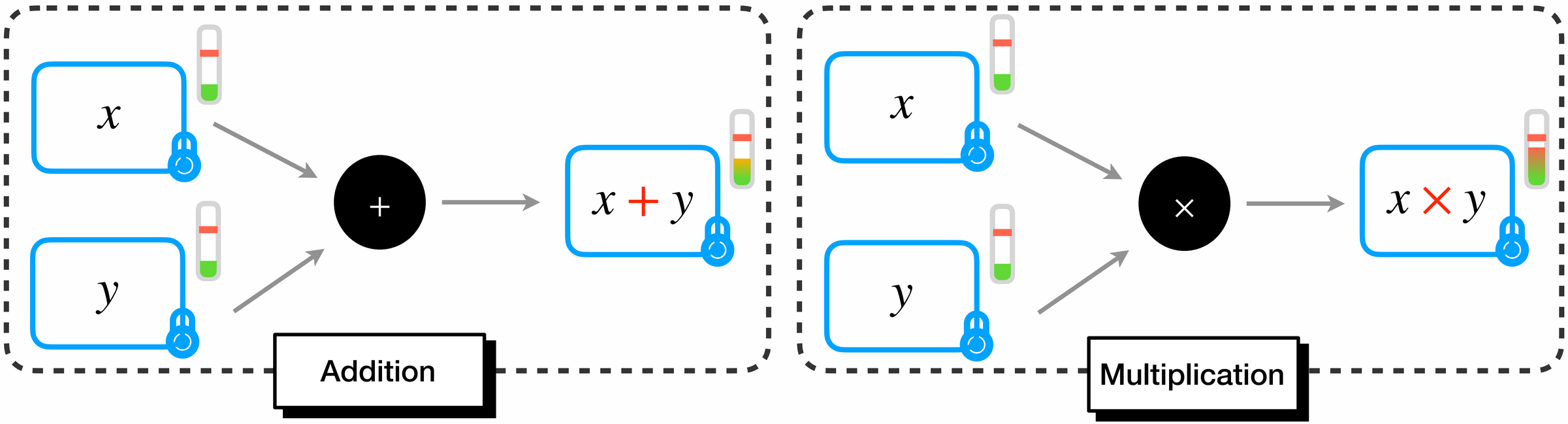
Parameter Optimization & Larger Precision for (T)FHE

ZAMA

Agenda

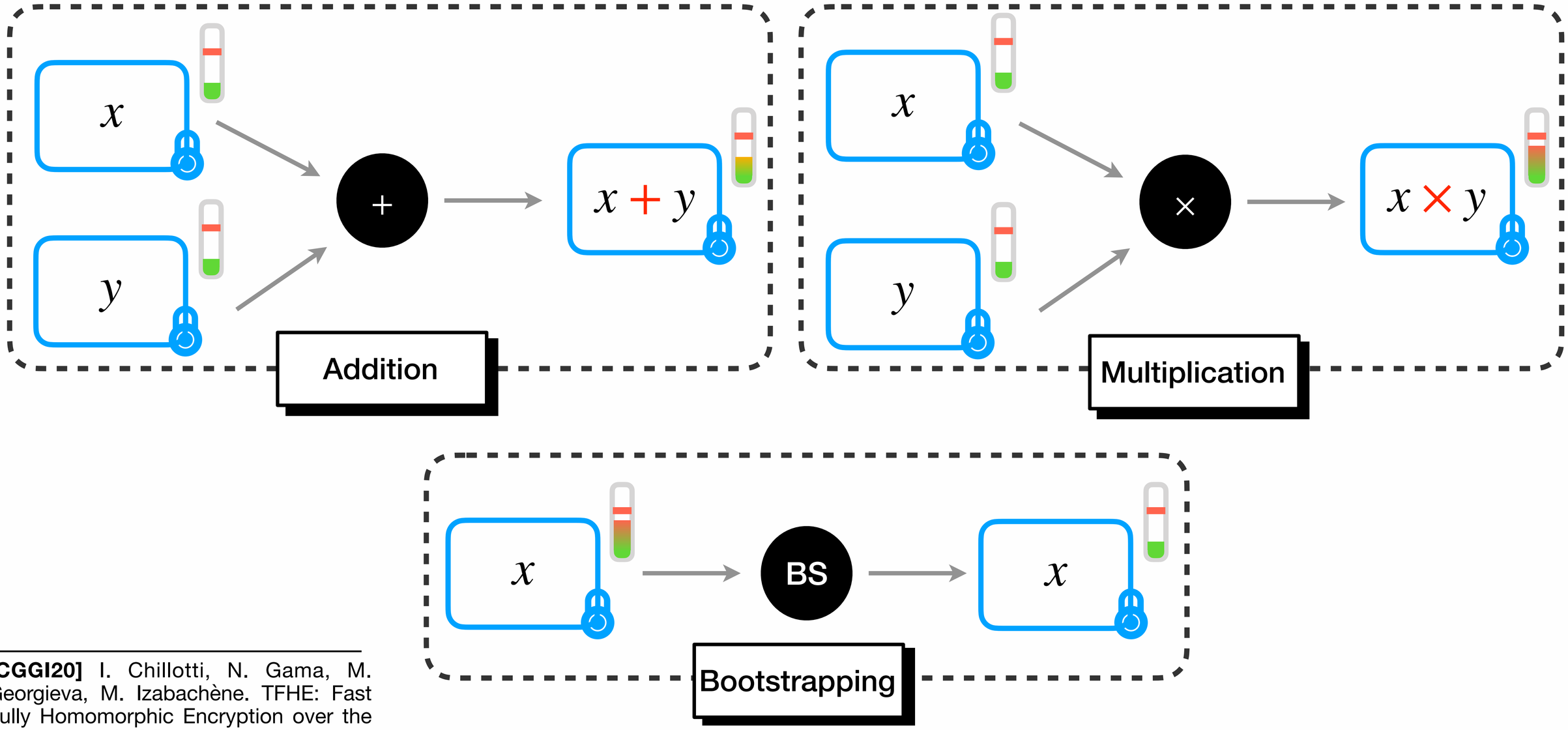
Introduction

FHE



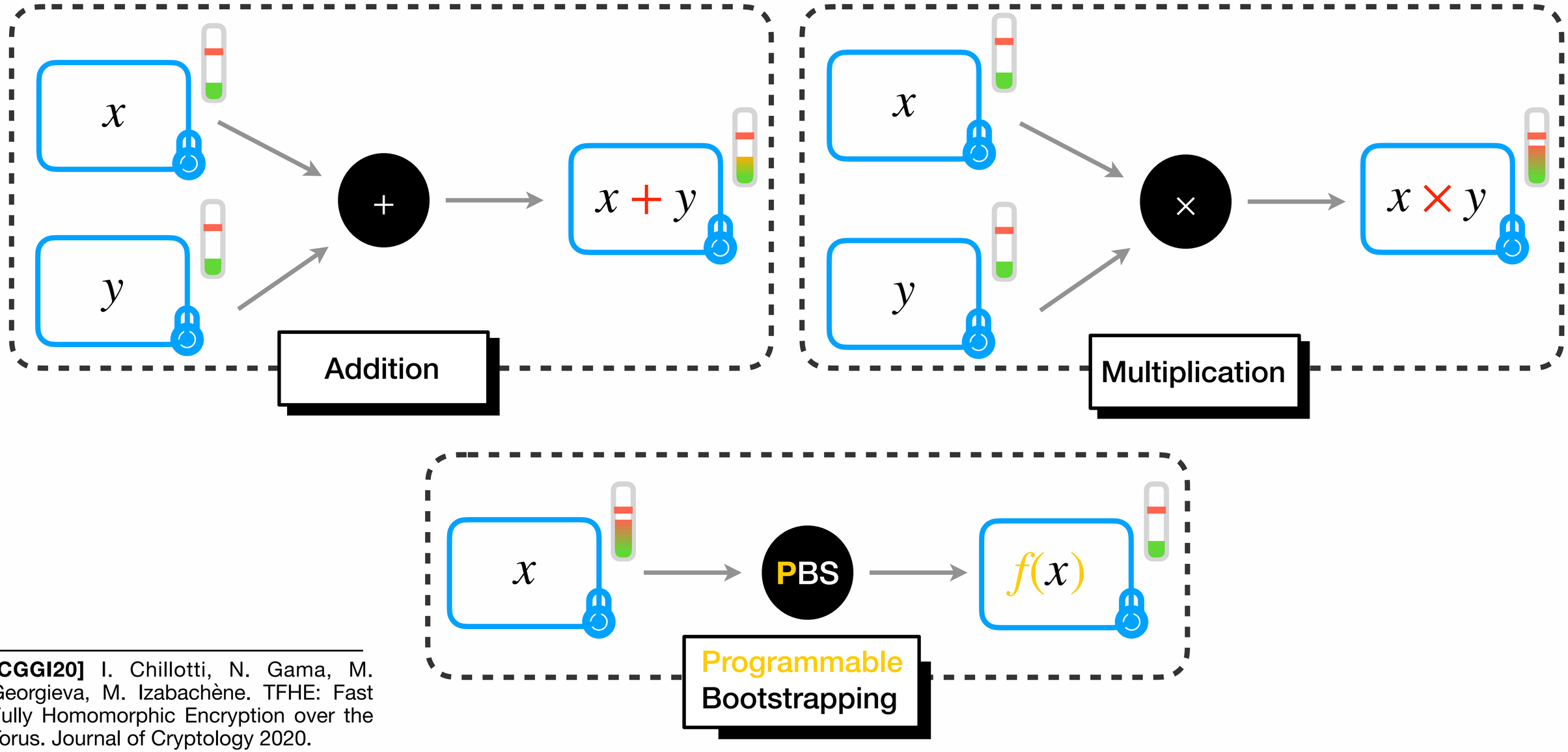
too much noise 🥵 $\Longrightarrow$ incorrect decryption

FHE



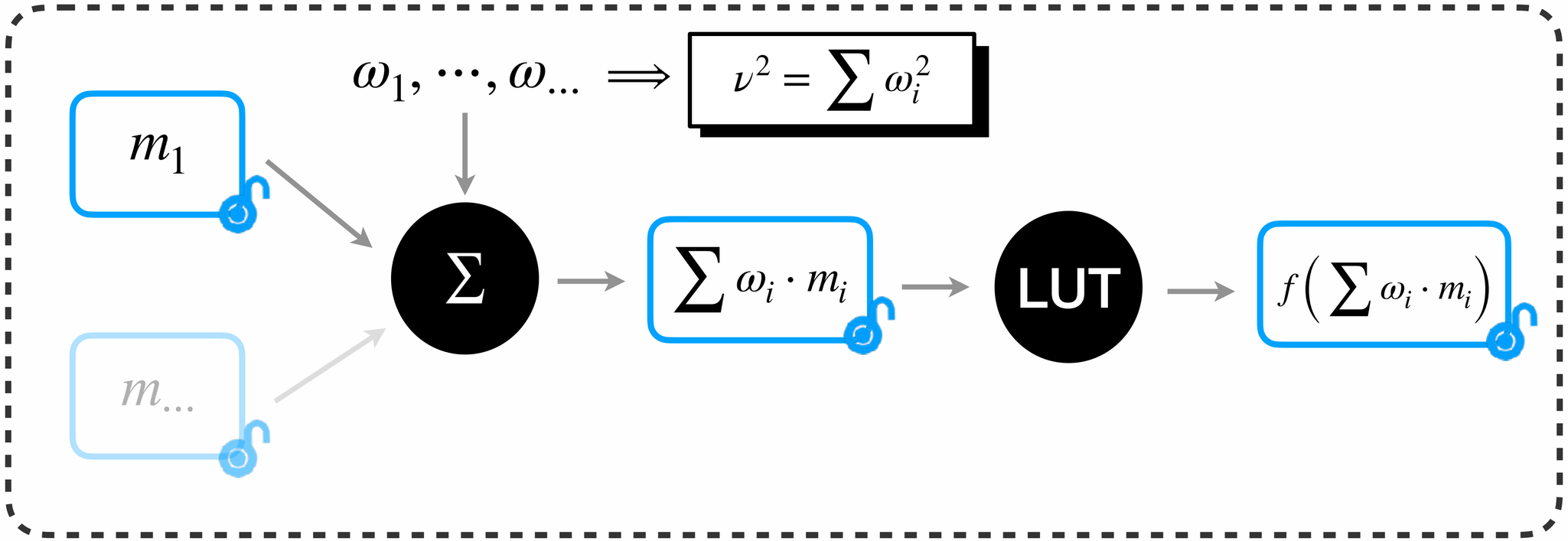
[CGGI20] I. Chillotti, N. Gama, M. Georgieva, M. Izabachène. TFHE: Fast Fully Homomorphic Encryption over the Torus. Journal of Cryptology 2020.

FHE



[CGGI20] I. Chillotti, N. Gama, M. Georgieva, M. Izabachène. TFHE: Fast Fully Homomorphic Encryption over the Torus. Journal of Cryptology 2020.

Plain Atomic Pattern



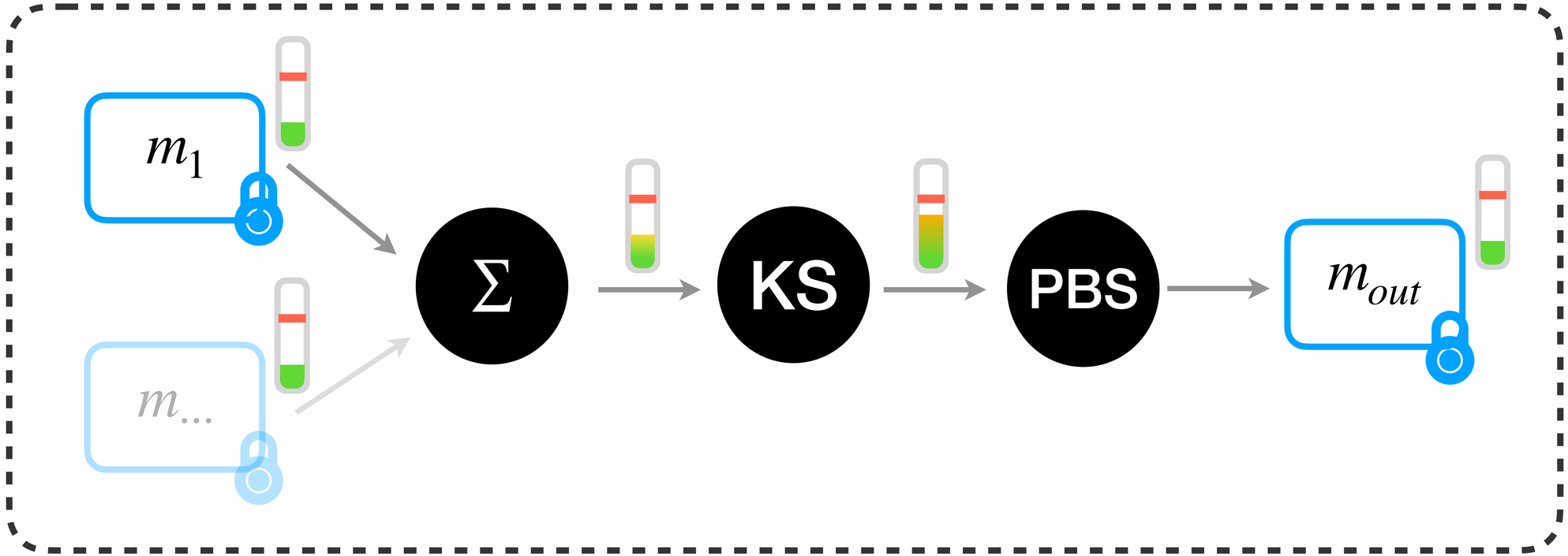
Symbolic Rewriting

Easy to transform a computation graph into a graph of atomic patterns

Recurrent Pattern

Enable simple analysis

CJP Atomic Pattern



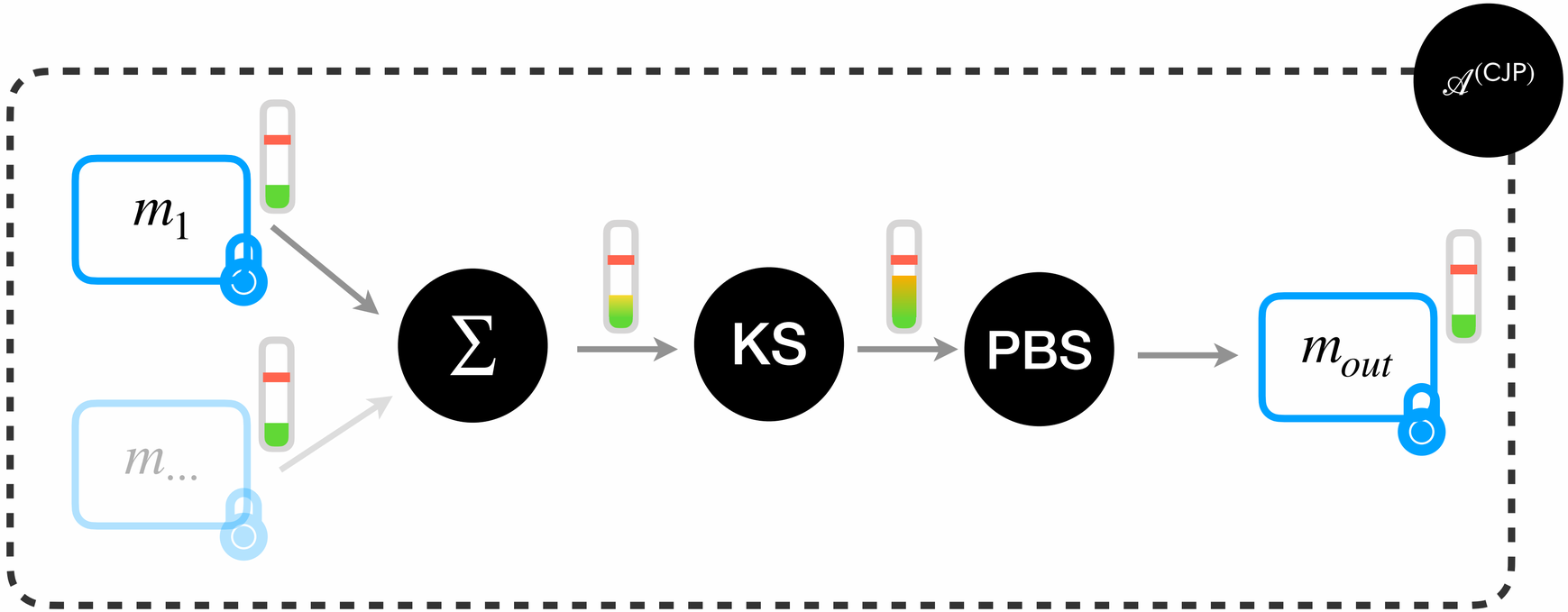
Leveled Operations

Keyswitching

Programmable Bootstrapping

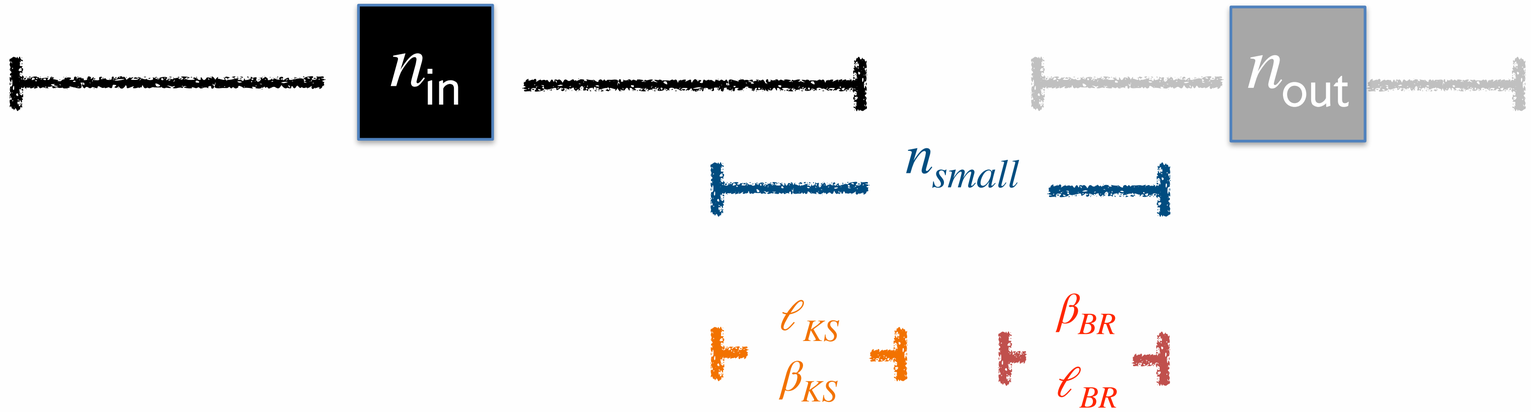
[CJP21] Ilaria Chillotti, Marc Joye, and Pascal Paillier. Programmable bootstrapping enables efficient homomorphic inference of deep neural networks. In CSCML 2021

CJP Atomic Pattern



Macro Parameters

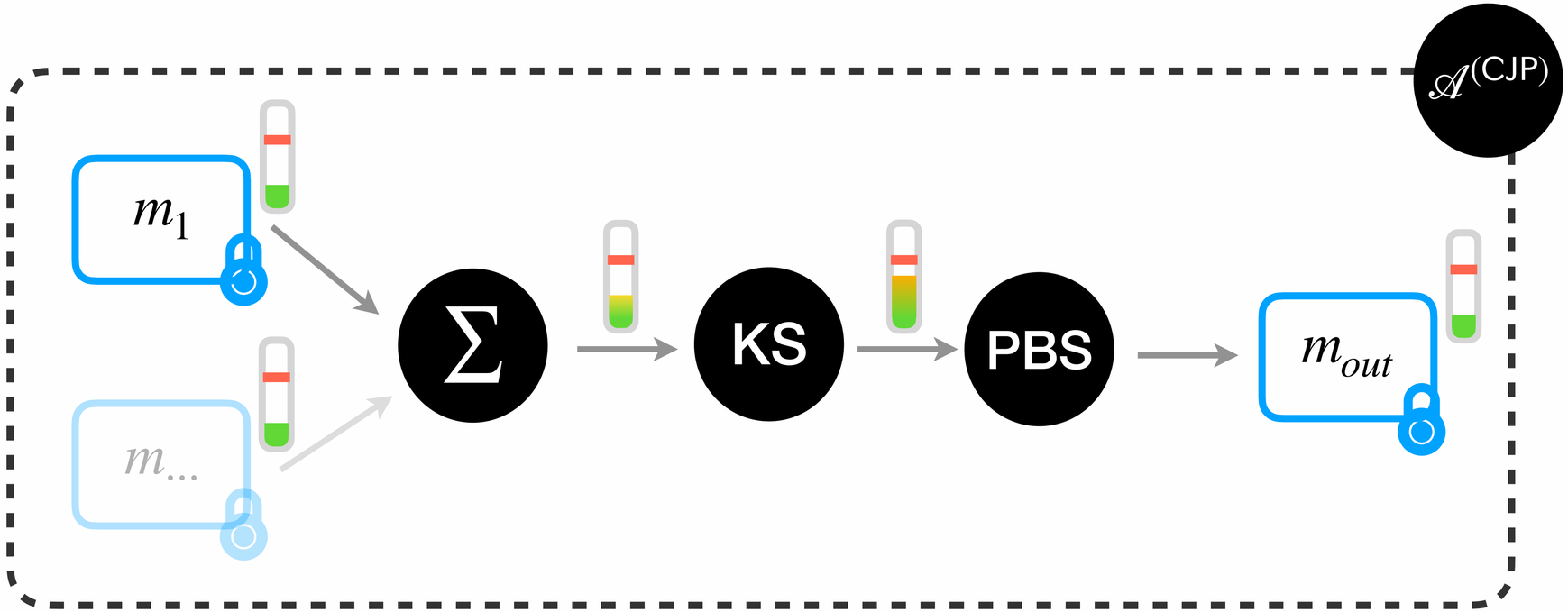
Micro Parameters



$\approx 2^{40}$

possible parameters

CJP Atomic Pattern



Macro Parameters

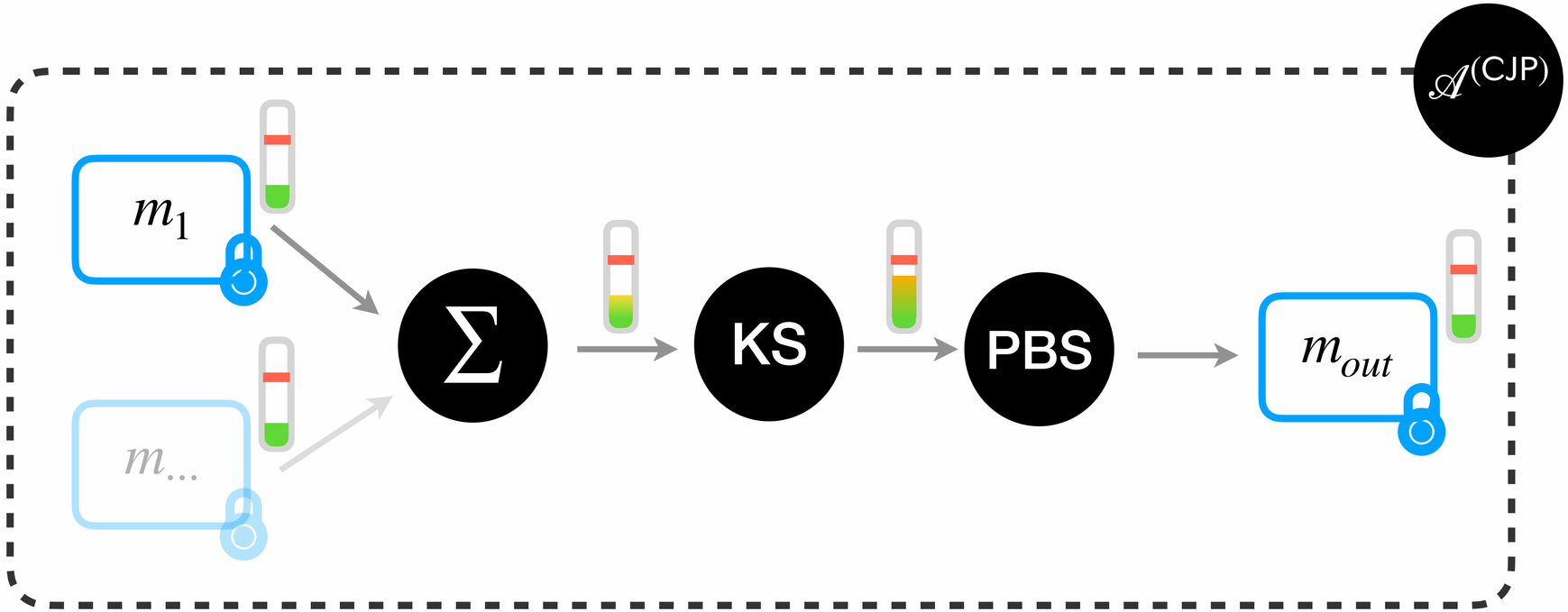
Micro Parameters

n_{in} k' N' n_{small} ℓ_{KS} β_{KS} β_{BR} ℓ_{BR}

$\approx 2^{40}$

possible parameters

CJP Atomic Pattern



Macro Parameters

Micro Parameters

$k \cdot N$ $\quad k' N'$

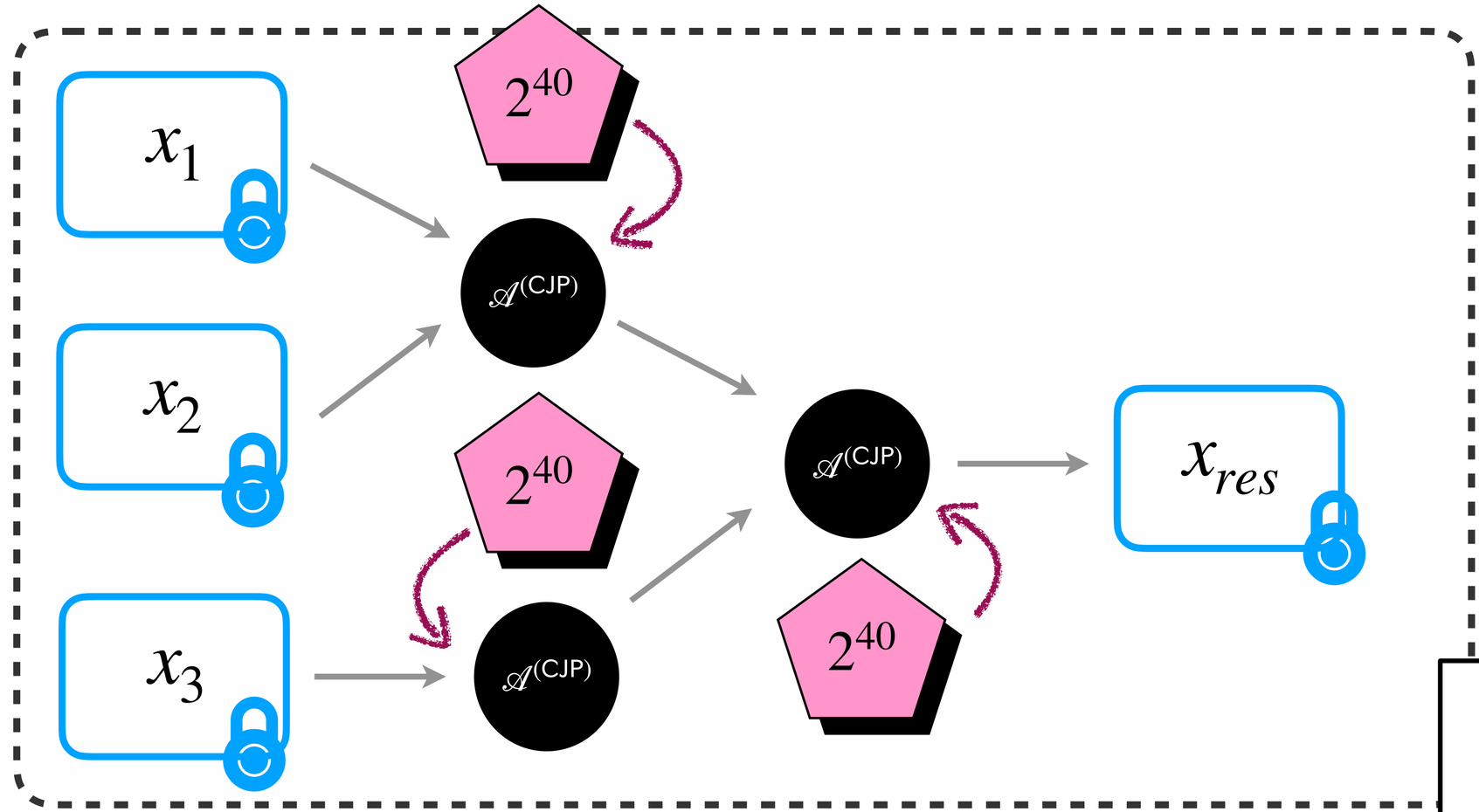
n_{small}

ℓ_{KS} β_{KS} $\quad \beta_{BR}$ ℓ_{BR}

$\approx 2^{40}$

possible parameters

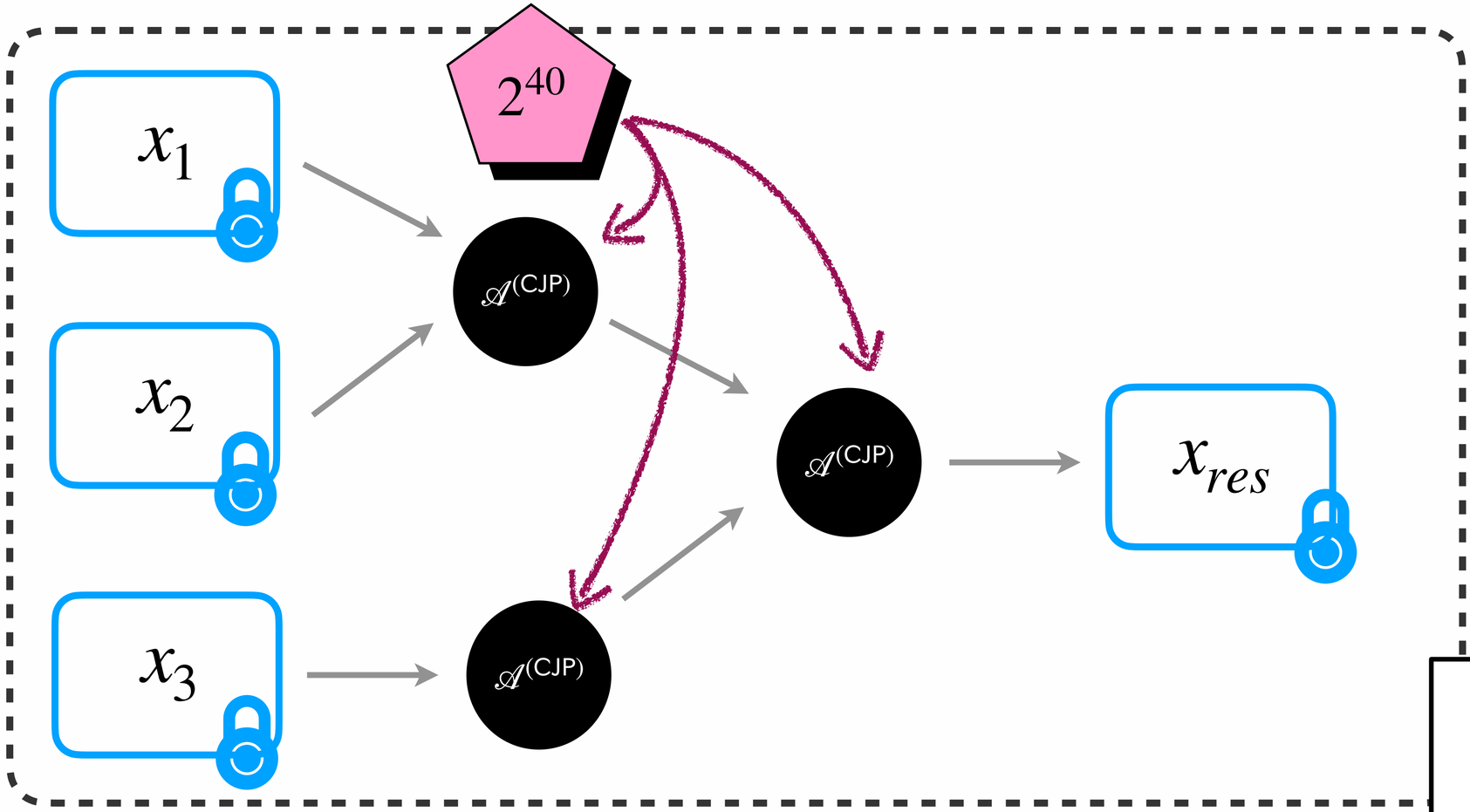
Graph of CJP AP



$\approx 2^{120}$

possible parameters

Graph of CJP AP

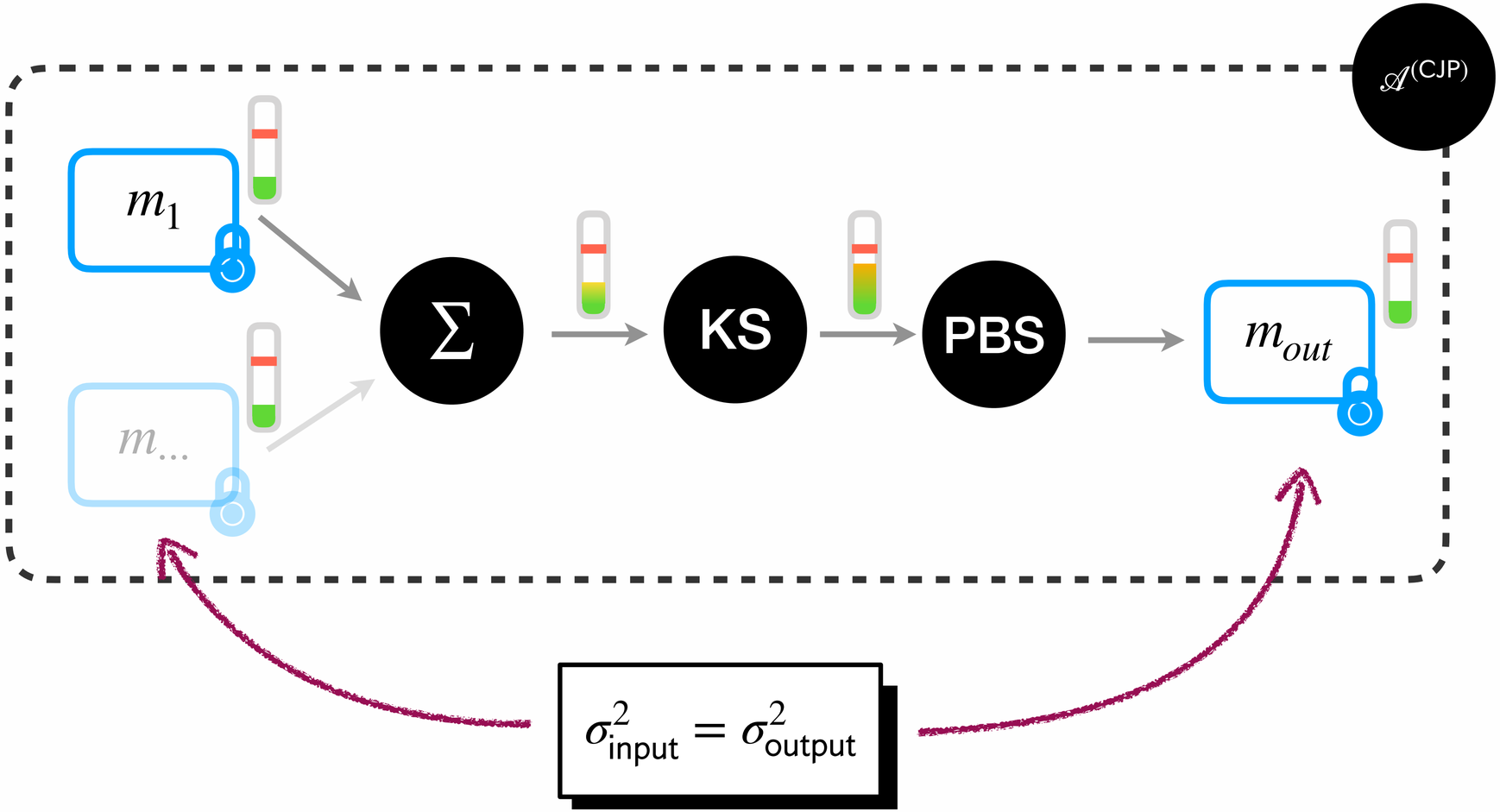


🧠 **1 Parameter set for the whole graph**

$\approx 2^{40}$

possible parameters

Graph of CJP AP



FHE Parameter Optimization

Overview

Overview: Goals

Security

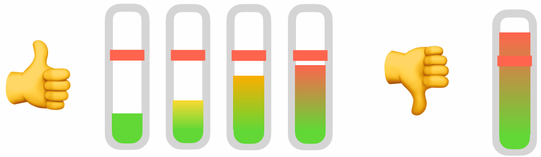
LWE dimension, Ciphertext Modulus

$$f : (n, \lambda, q) \mapsto \sigma_{\text{enc}}$$

Security level, Minimal standard deviation

→ Using the **lattice estimator**

Correctness



→ **Noise Model** to track the noise along the computation

Efficiency

→ **Cost Model** as a surrogate of the execution time

Overview: Problem

Let $\mathcal{G} = \{A_i\}_{i \in I}$

$$\min \text{Cost}(\mathcal{G}) \quad \text{s.t.} \begin{cases} \forall i \in I, \text{Noise}(A_i) \leq t^2 \\ \sigma_{\text{enc}} = f(n, \lambda, q) \end{cases}$$

up to a given p_{fail}

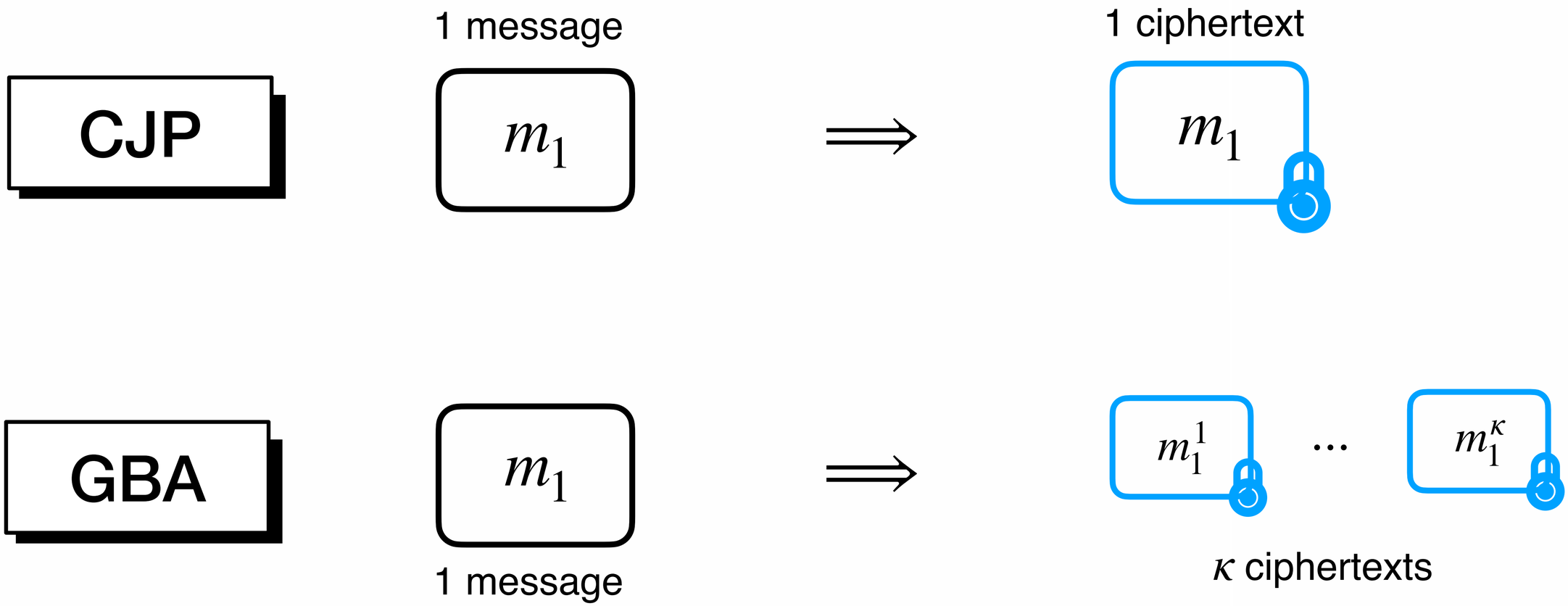
Noise bound

λ bits of security

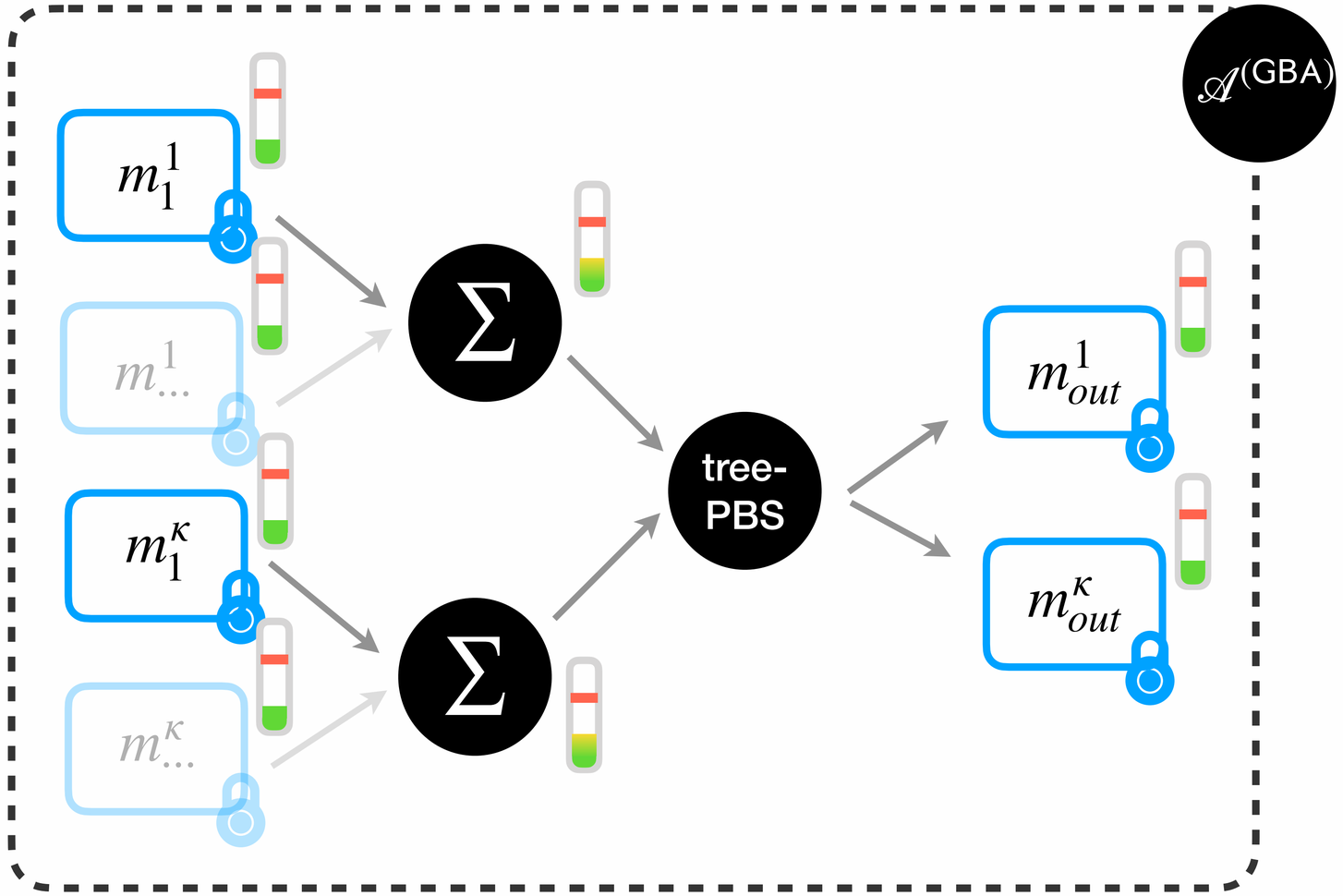
FHE Parameter Optimization

GBA Atomic Pattern

Encoding


CJP
1 message
m_1
$\Longrightarrow$
1 ciphertext
m_1
GBA
m_1
1 message
$\Longrightarrow$
m_1^1
...
m_1^κ
κ ciphertexts

GBA Atomic Pattern



$$\approx 2^{52}$$

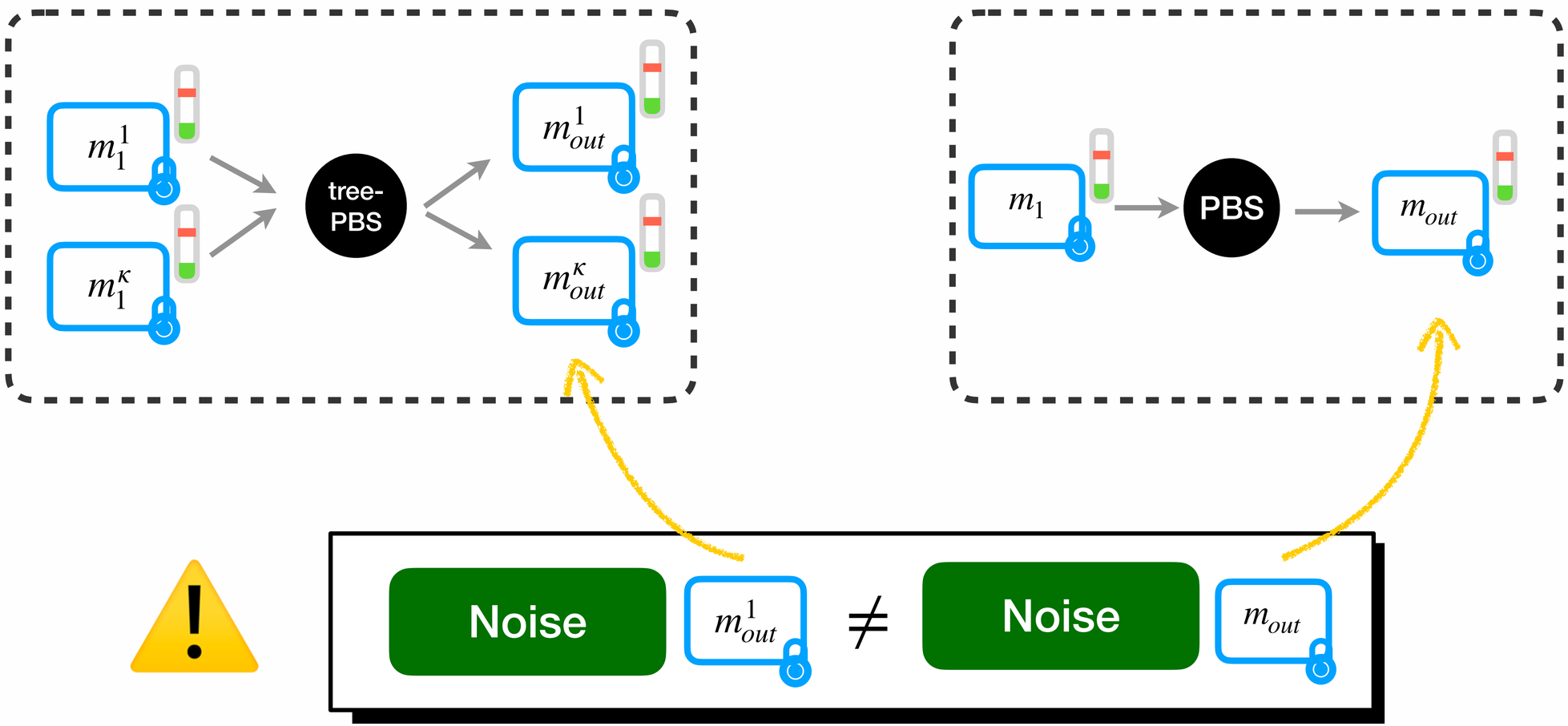
possible parameters

[GBA21] A. Guimaraes, E. Borin, D. Aranha. Revisiting the functional bootstrap in TFHE. IACR Transactions on Cryptographic Hardware and Embedded Systems

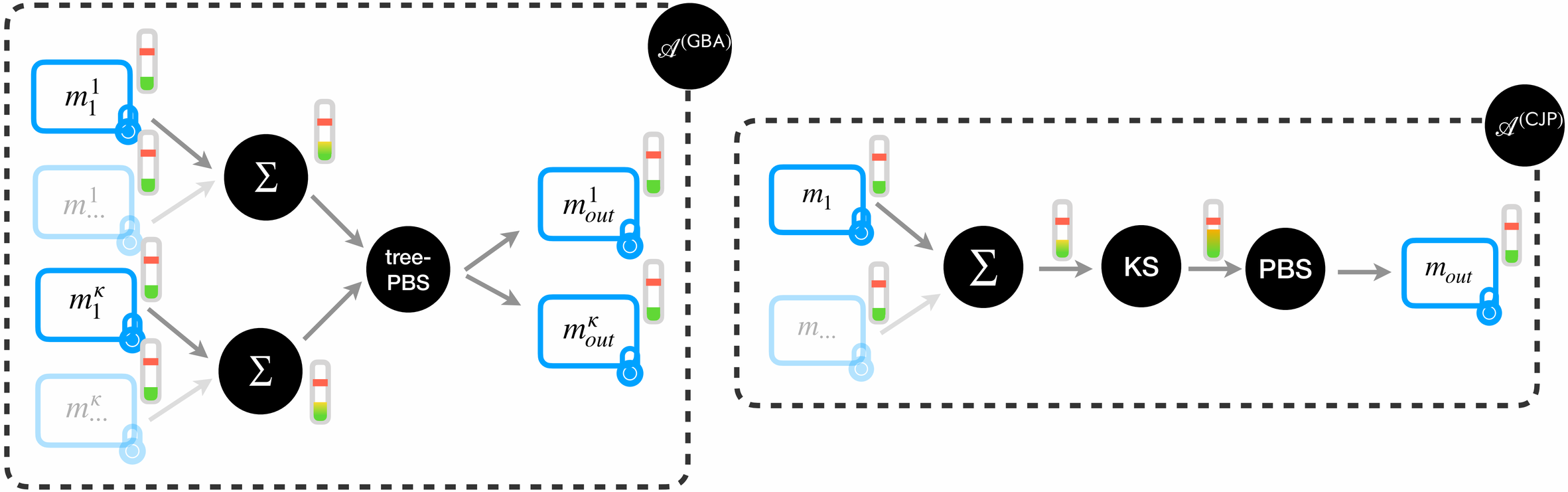
FHE Parameter Optimization

CJP vs GBA

CJP vs GBA

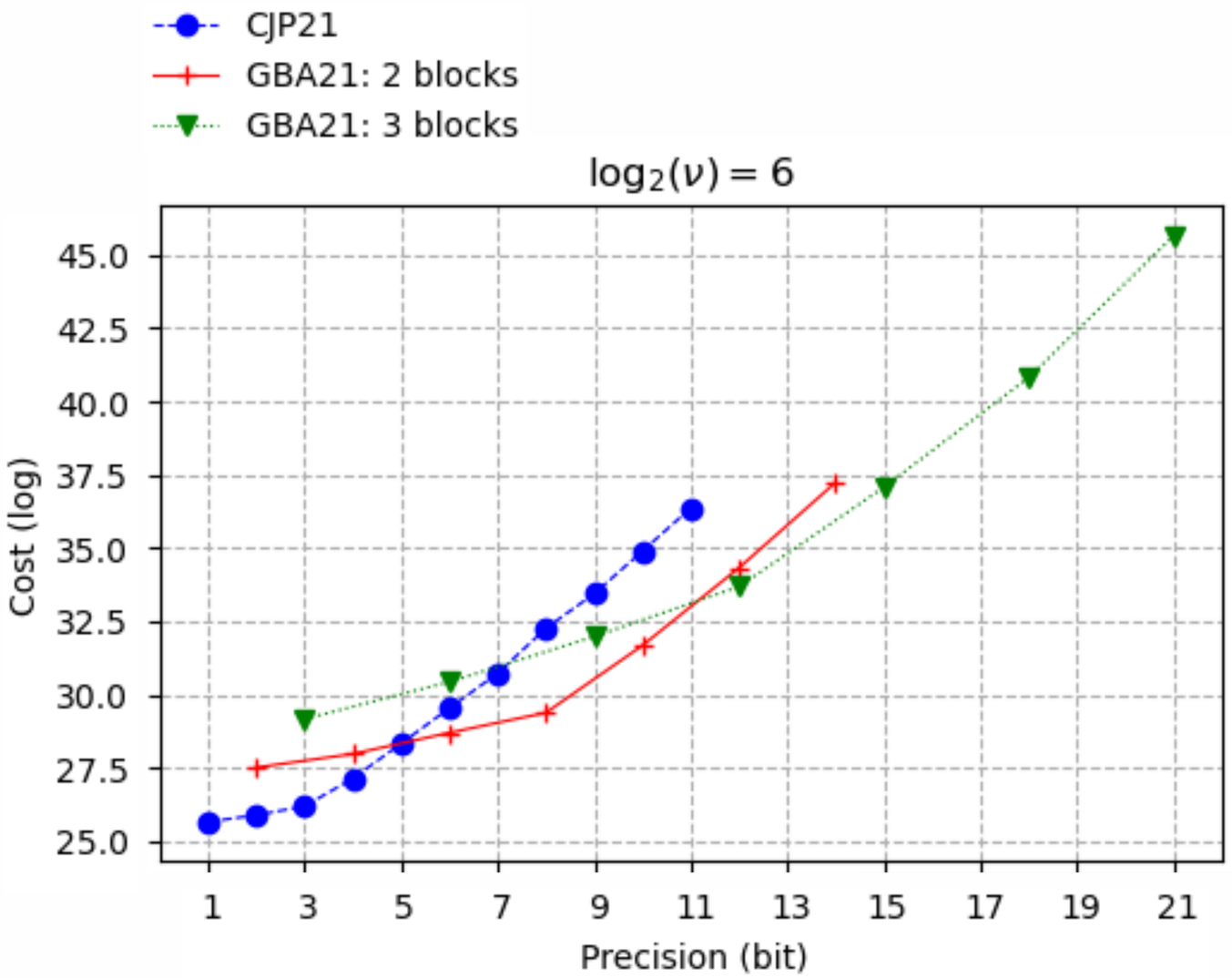

m_1^1
m_1^κ
tree-PBS
m_{out}^1
m_{out}^κ
m_1
PBS
m_{out}
Noise
m_{out}^1
≠
Noise
m_{out}

CJP vs GBA



Context-aware comparison

CJP vs GBA



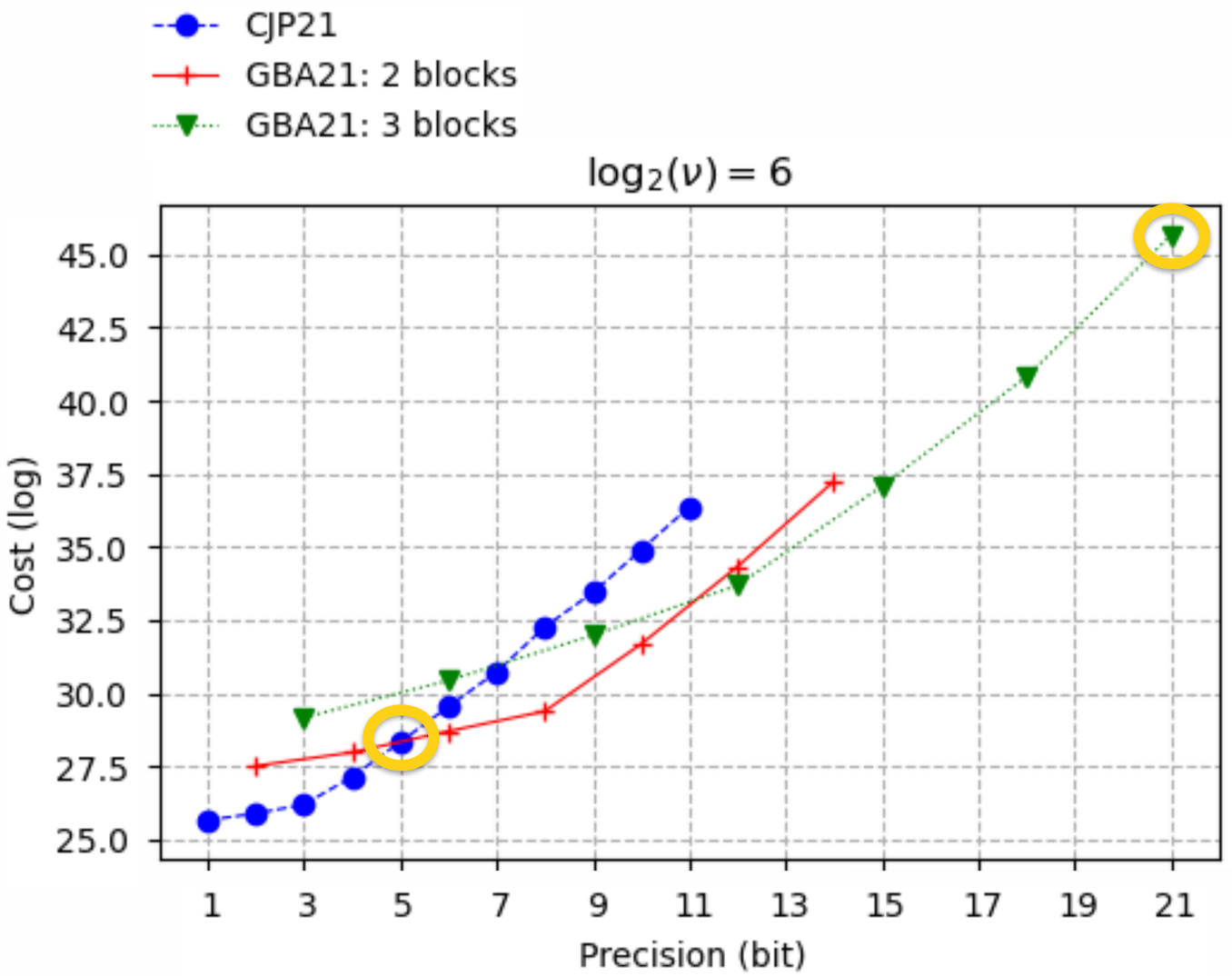
Efficient alternative to TFHE PBS above **5 bits**

Allows **bigger** precision (up to **21 bits**)

Large precision are **very costly**

$$\text{Cost}(21\ bits) \approx 2^{17} \cdot \text{Cost}(5\ bits)$$

CJP vs GBA



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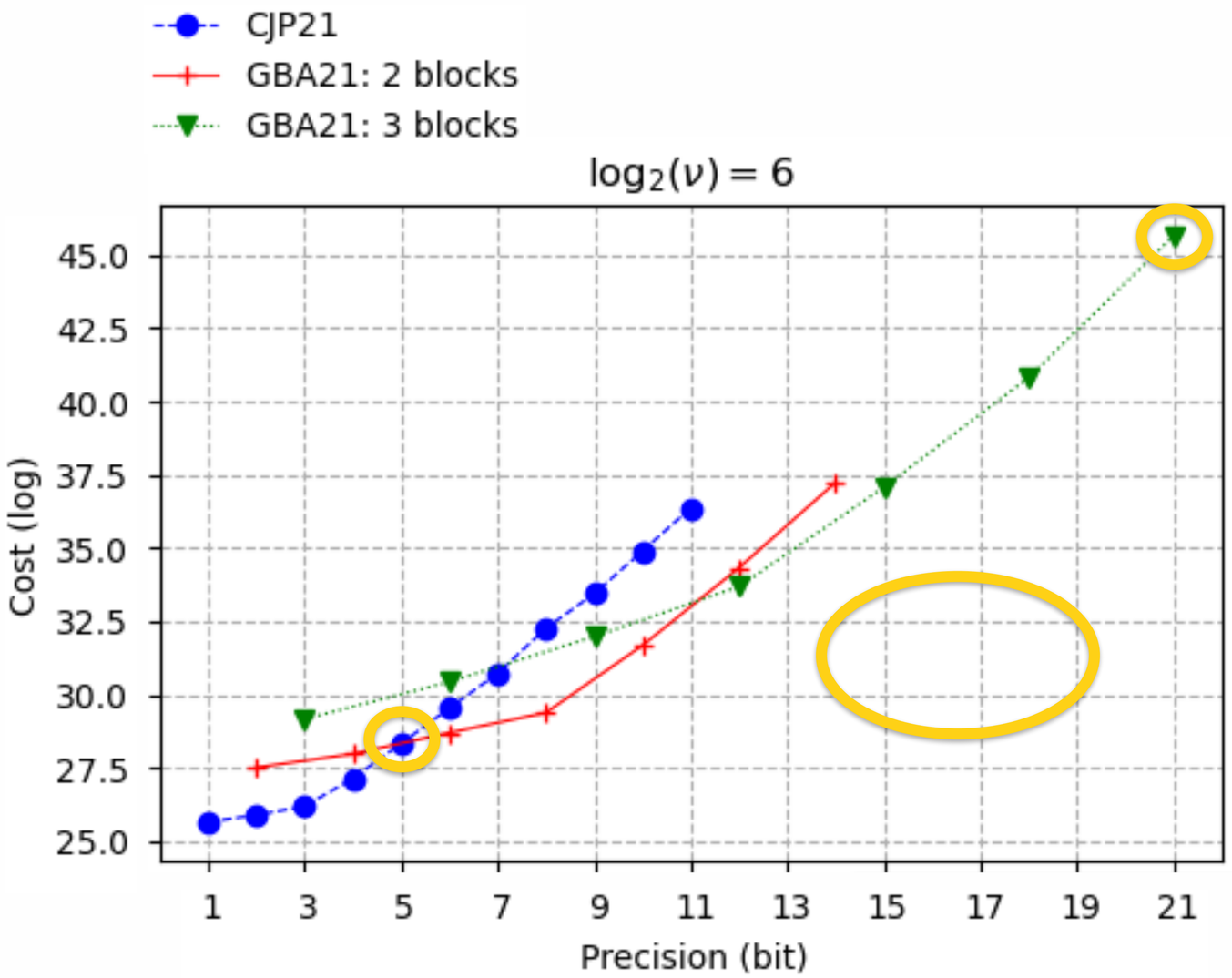
CJP vs GBA



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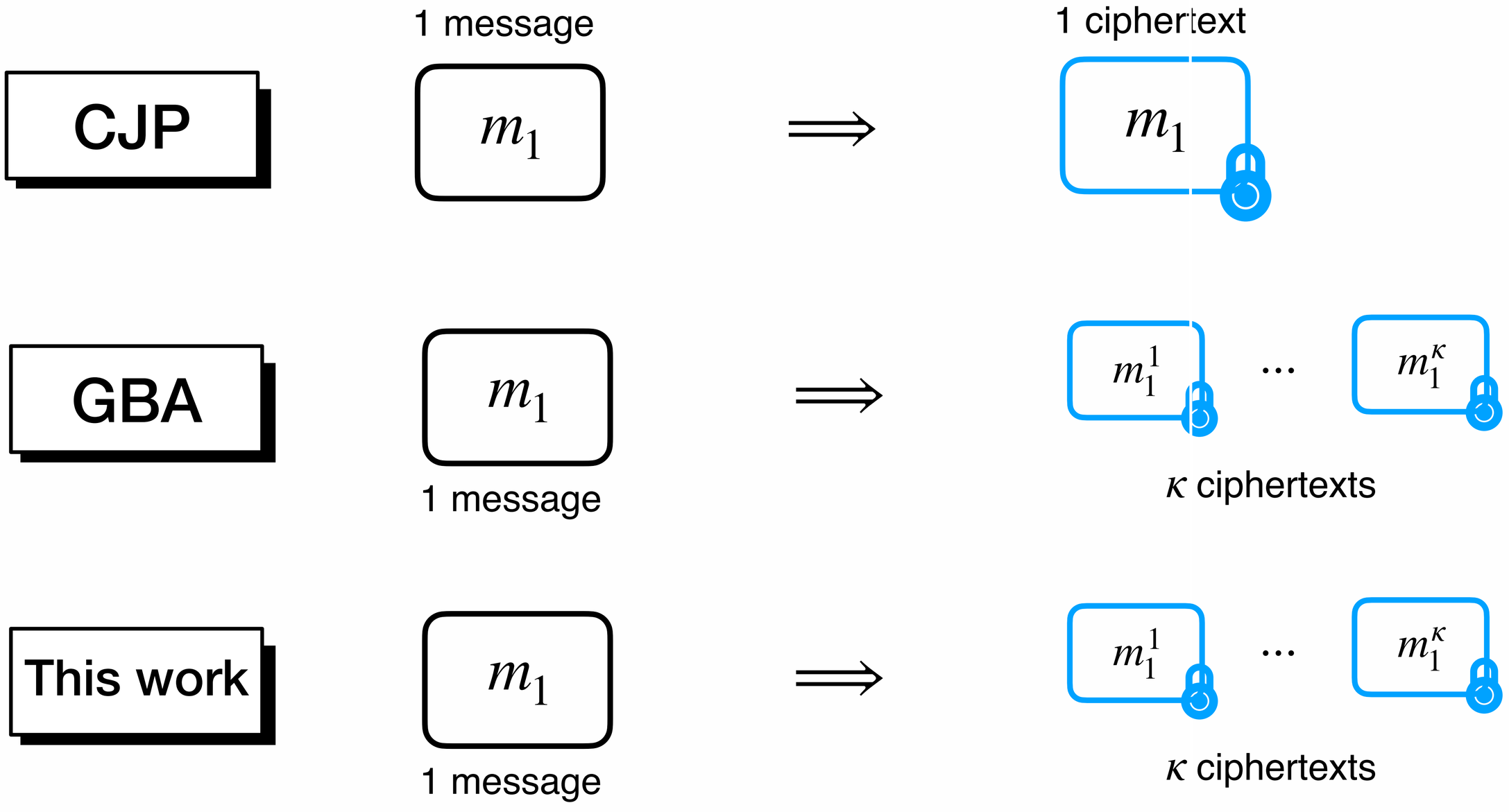
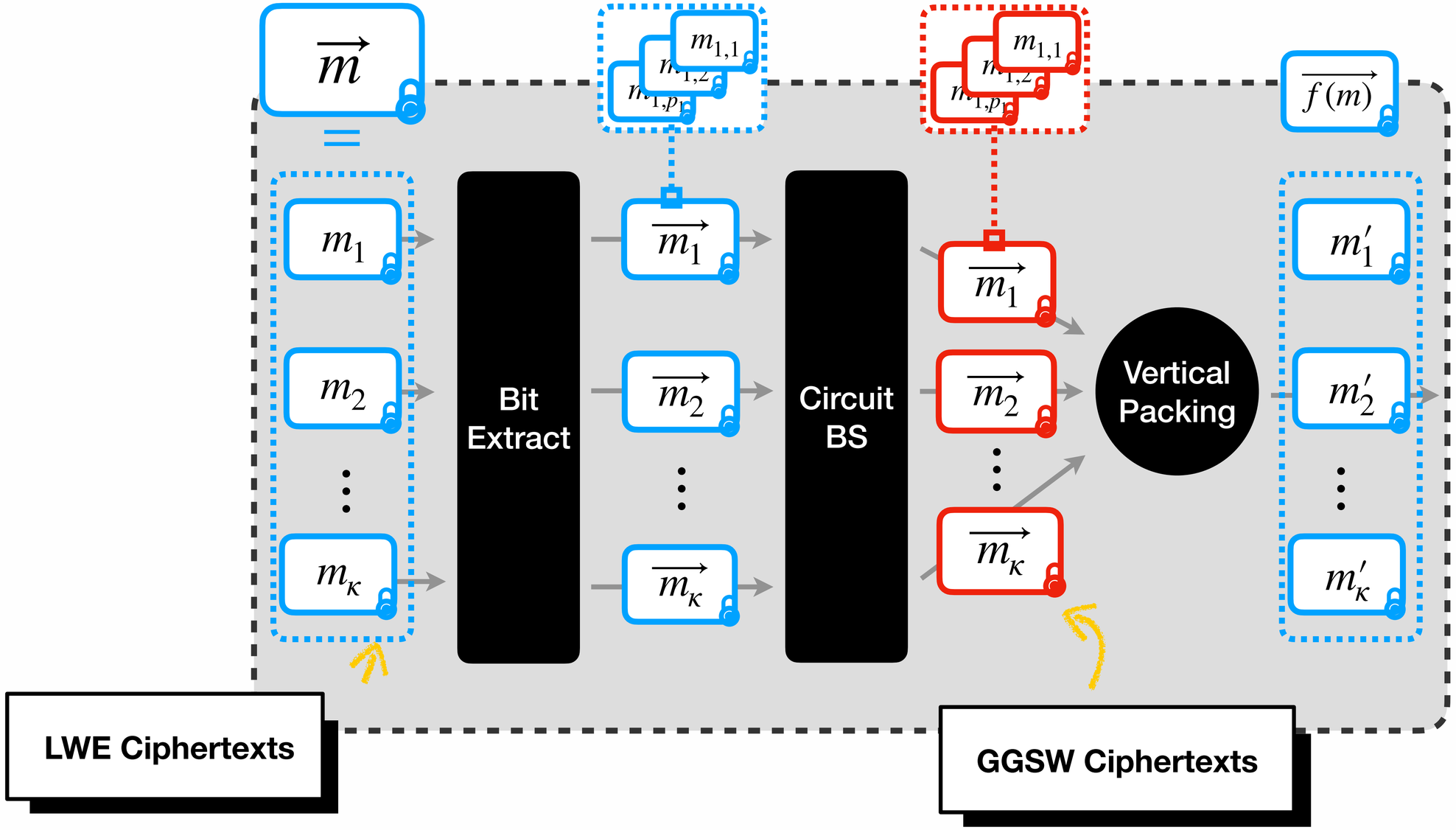
WoP-PBS

Overview

Encoding


CJP
1 message
m_1
1 ciphertext
m_1
GBA
m_1
1 message
m_1^1 ... m_1^κ
κ ciphertexts
This work
m_1
1 message
m_1^1 ... m_1^κ
κ ciphertexts

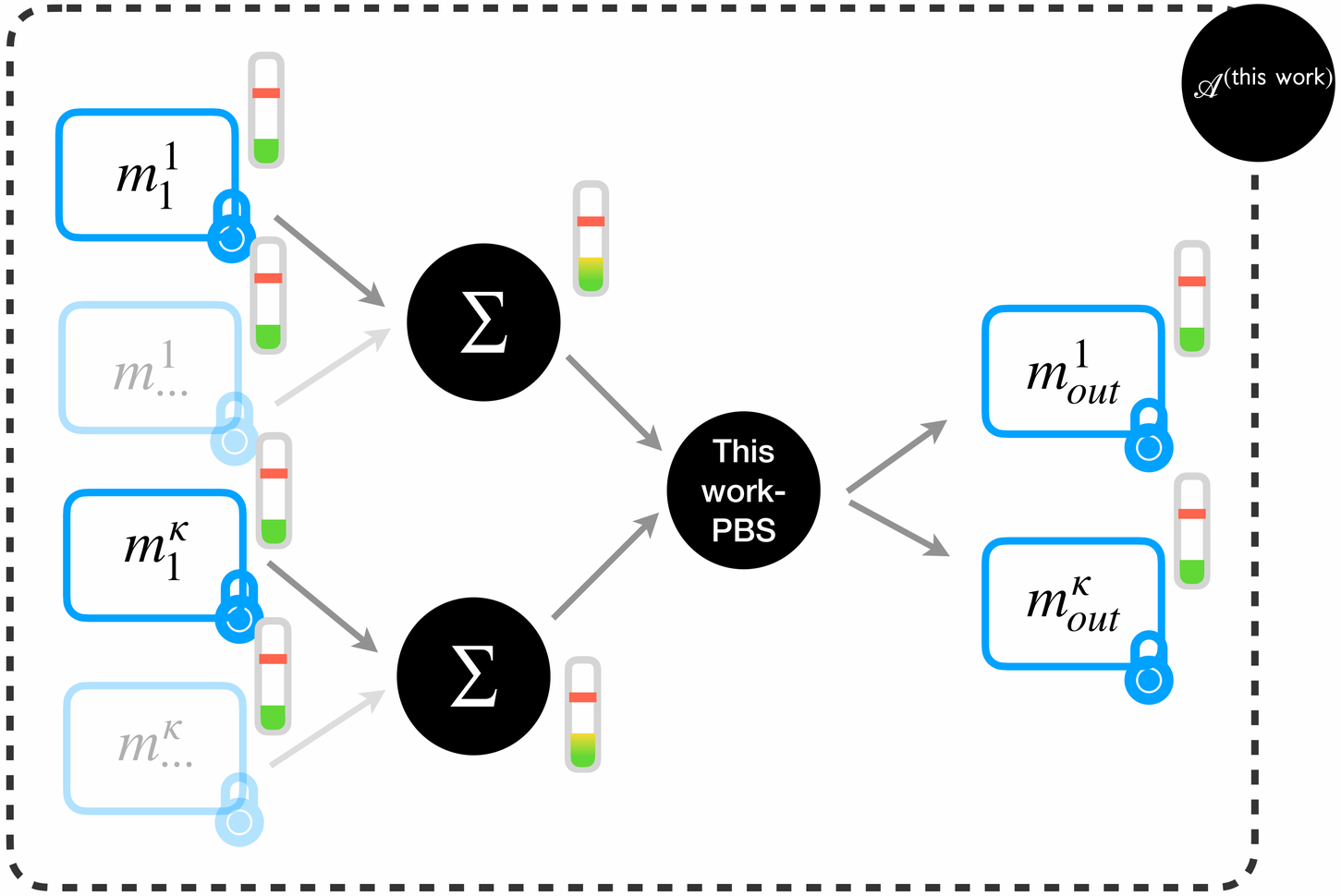
New WoP-PBS


$\overrightarrow{m}$
=
m_1
m_2
m_κ
Bit Extract
$m_{1,1}$
$m_{1,2}$
m_{1,p_1}
$\overrightarrow{m_1}$
$\overrightarrow{m_2}$
$\overrightarrow{m_\kappa}$
Circuit BS
$m_{1,1}$
$m_{1,2}$
m_{1,p_1}
$\overrightarrow{m_1}$
$\overrightarrow{m_2}$
$\overrightarrow{m_\kappa}$
Vertical Packing
$\overrightarrow{f(m)}$
m'_1
m'_2
m'_κ
LWE Ciphertexts
GGSW Ciphertexts

WoP-PBS

Comparisons

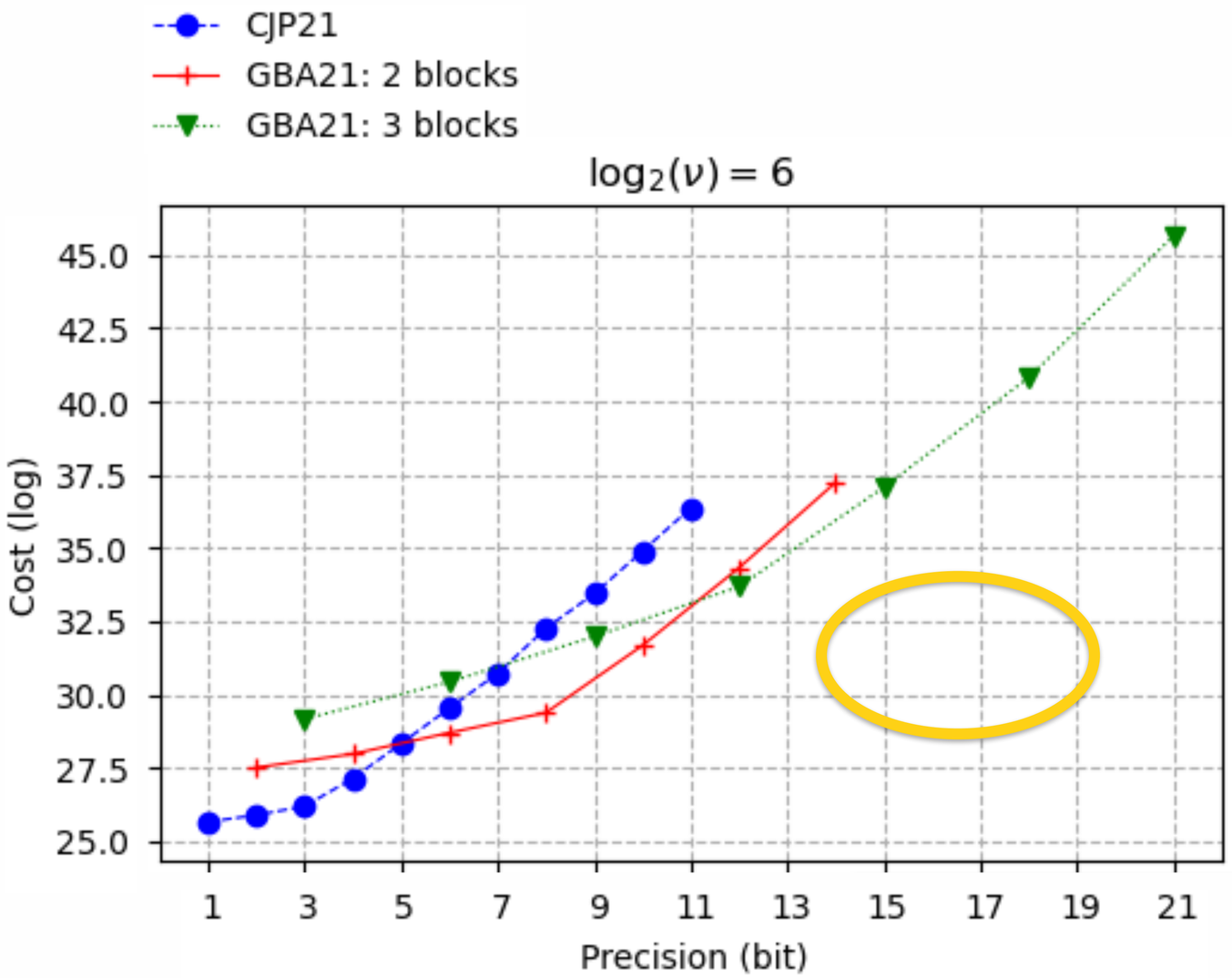
This work Atomic Pattern



$\approx 2^{64}$

possible parameters

CJP vs GBA



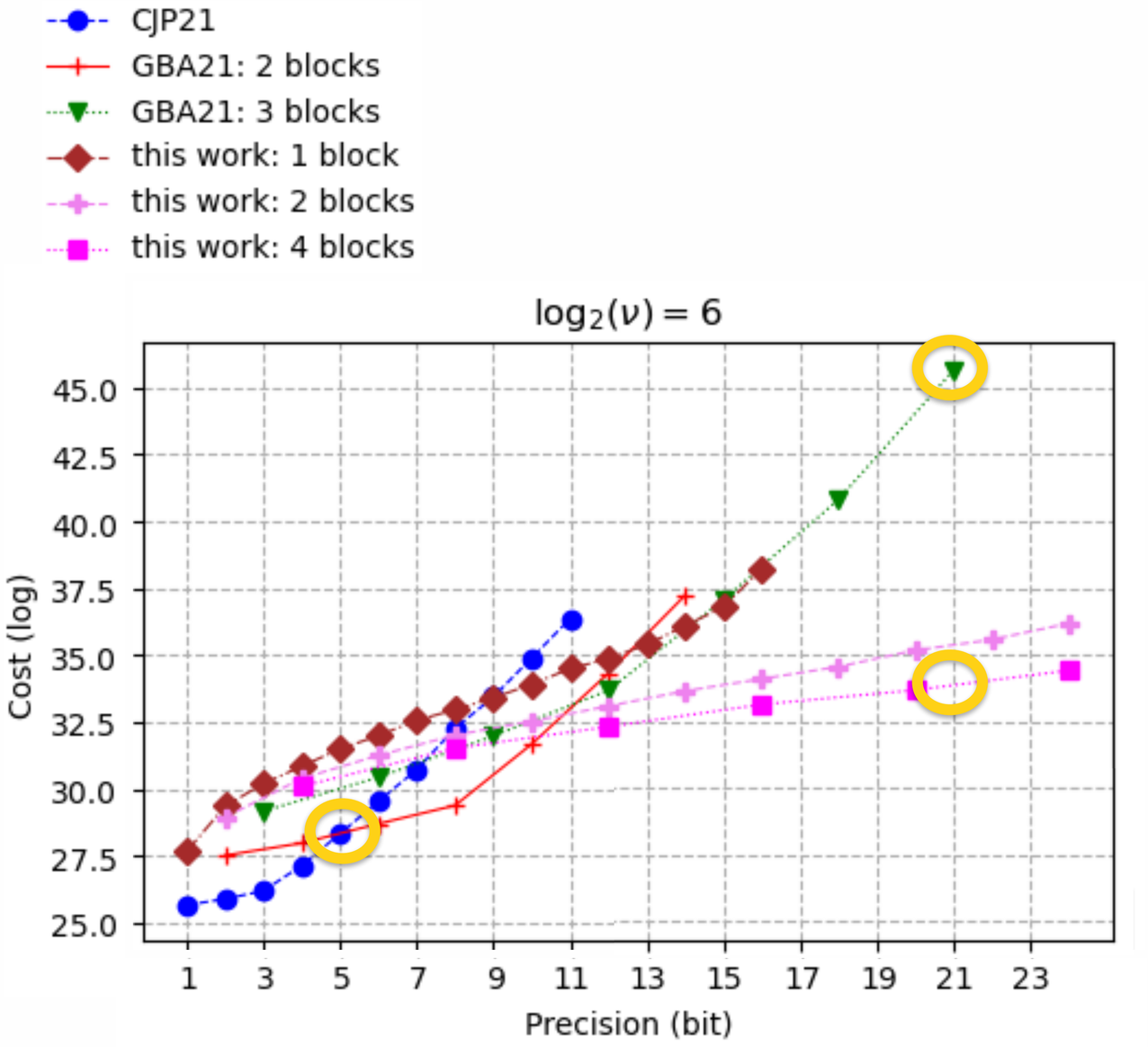
Efficient alternative to TFHE PBS above **5 bits**

Allows **bigger** precision (up to **21 bits**)

Large precision are **very costly**

$$\text{Cost}(21\ bits) \approx 2^{17} \cdot \text{Cost}(5\ bits)$$

CJP vs GBA vs this work



Efficient alternative to GBA-PBS above **10 bits**

Allows **bigger** precision (up to **24 bits**)

Large precision are **less costly**

$$\text{Cost}(21\ bits) \approx 2^{17} \cdot \text{Cost}(5\ bits)$$
$$\approx 2^{12} \cdot \text{Cost}(5\ bits)$$

Conclusion

Other results

Other results

Large Integers

CRT, radix, hybrid encoding

WoP-PBS Analysis

LMP, this work

Failure Probability

AP and graph level

KS Position

CJP, CGGI, KS-free

PBS Insertion

In Dot Product

Several KSK/BSK

CJP

Conclusion

Future Work

Future Work

Better Cost Model

In the paper: algorithmic complexities

Better Noise Model

In the paper: from [CLOT21]

Multi Parameter Set

In the paper: only one parameter set

Graph Comparison

Real use cases

Thank you.

ZAMA

Contact and Links

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samuel.tap@zama.ai

zama.ai

Github

Community links

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Bibliography

[CGGI20] I. Chillotti, N. Gama, M. Georgieva, M. Izabachène. TFHE: Fast Fully Homomorphic Encryption over the Torus. Journal of Cryptology 2020.

[CJP21] Ilaria Chillotti, Marc Joye, and Pascal Paillier. Programmable bootstrapping enables efficient homomorphic inference of deep neural networks. In CSCML 202

[CLOT21] I. Chillotti, D. Ligier, J-B Orfila, and S. Tap. Improved programmable bootstrapping with larger precision and efficient arithmetic circuits for tfhe. In ASIACRYPT 2021

[GBA21] A. Guimaraes, E. Borin, D. Aranha. Revisiting the functional bootstrap in TFHE. IACR Transactions on Cryptographic Hardware and Embedded Systems

[LMP21] Zeyu Liu, Daniele Micciancio, and Yuriy Polyakov. Large-precision homomorphic sign evaluation using fhew/tfhe bootstrapping. Cryptology ePrint Archive, Report 2021/1337